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Analysis of A Practical Application of Geodetic Methods of Robust Estimation in a Vessel Positioning Based on Radar Observations

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Abstract. This paper analyses whether the application of robust estimation methods can prevent incorrect vessel positioning, thereby improving navigation safety. Empirical analyses were carried out assuming that the observational vector $\mathbf{y}$ comprises observations acquired from coastal radar stations operating within the "VTS Gdańsk Bay" system. Particular attention was paid to a situation in which one of observations has a gross error. Numerical tests were carried out considering several variants of the number of redundant observations. Since the significance of the subject matter in this study arises from the dynamism of position changes of a measurement item, measurements cannot be repeated at the same position. For the same reason, it is very important to choose the optimum method of working out the observations. The results of the robust adjustment of radar observations were compared with the results of the conventional least squares method.

Keywords: geodetic adjustment, robust estimation, radar navigation.

Conference topic: Technologies of geodesy and cadastre.

Introduction

Vessel tracking systems (VTS) are established in basins with intensive ship traffic and the presence of navigation hazards creates a considerable risk of collision or other maritime disaster. A VTS is established by a marine administrative body of the state with sea access in order to improve navigation safety, environmental protection or to streamline vessel traffic. Depending on the needs, the basin shape and vessel traffic stream, a VTS can include a port and an approach to it (e.g. Rotterdam Port) or an approach to several ports situated close to each other (e.g. Gdańsk Bay), etc. The area covered by the system is usually serviced by the control centre. The main task of each centre is to analyse the navigational situation in the area and VTS centre operators must have complete and precise information on the position of the vessels moving in the basin in which the VTS is established. It is extremely important for an operator to have as precise a position as possible together with an assessment of its quality, for example using satellite systems (see Specht *et al.* 2015; Specht, Rudnicki 2016). The authors of this paper have analysed modern analytical methods to streamline the work of a VTS operator. The effects of one of the work stages are presented in this paper. VTS Gdańsk Bay, where the theoretical assumptions were verified, has been operating since 1 May 2003. It serves VTS system users and supervises navigation safety within the area of its responsibility. The VTS Gdańsk Bay system was accepted in October 2007 and approved formally during the 83rd session of Maritime Safety Committee IMO. As decided by the IMO's Maritime Safety Committee, the regulation became effective on 1 May 2008. The system has been employing:

- 5 coastal radars deployed at the following sites: the Hel lighthouse, the Gdynia harbour master's office, the Gdańsk harbour master's office, Górki Zachodnie and the Krynica Morska lighthouse;
- a system of automatic vessel identification using the base station at the Hel lighthouse;
- radio direction finder (RDF).

Information on the position of a vessel in the system is acquired from several coastal radar stations from which redundant observations are acquired. To use them effectively, the authors propose that modern, non-conventional methods of observation alignment should be employed, which are known in contemporary geodesy. The least squares method is recommended in working out observations in such issues; it is one of a broad family of M-estimation methods. In the issue under discussion. it is very important that a process of determining the coordinates of a vessel should take into account those measurement results which are contaminated with unavoidable random errors, which are present in each measurement process (see e.g. Berné, Baselga 2005). However, in navigational practice, there is a risk of the appearance of additional interference in the measurements. The literature refers to such interference as gross measurement errors and they can usually arise from an erroneously read out measurement result, temporary changes of the parameters of the measurement environment or improper calibration of measurement devices. Measurement results with gross errors are referred to as outlying observations.

Therefore, the estimation of a vessel's position by the least squares method does not guarantee reliable information on the vessel's position if a measurement result appears in the observation vector which has a gross error. This is a result of the high sensitivity of the LS method to outlying observations (see e.g. Guo *et al.* 2010). Therefore, correct results of an estimation can be obtained only when measurement results with a gross error are eliminated with a set of observations. The most popular methods of detection and elimination of outlying observations include those based on statistical tests (see e.g. Baarda 1968; Pope 1976). However, practical application of statistical tests for detecting outlying observations in navigational measurement structures is very difficult to accomplish. This results mainly from high dynamism of vessel positions. Therefore, the response time for a manoeuvre to change the position of a vessel should be minimised, especially in areas where navigation may be dangerous or where vessel traffic is dense. The application of robust estimation methods is another method of detection of gross errors which can be considered in navigational measurement structures. The theory of estimation provides several methods of determining robust estimators, including the methods of L-estimation (see e.g. Huber 1981; Koenker, Portnoy 1987), R-estimation (see e.g. Rousseeuw, Verboven 2002; Duchnowski 2010) and robust M-estimation (see e.g. Huber 1981; Yang *et al.* 2002). A specific generalisation of M-estimation, i.e. M_{split} estimation, can also be regarded as a method of robust estimation (see e.g. Wiśniewski 2009; Ge *et al.* 2013; Zienkiewicz 2014). Robust methods of observation adjustment, included in the class of M-estimation, are particularly important from the perspective of navigational measurement structures. The effect of outlying observations on the final result of estimation in these methods is eliminated or at least limited by application of the appropriate weight function. Such an approach to working out observations acquired from coastal radar stations has been proposed and discussed in detail in previous papers (Czaplewski 2004; Świerczyński, Czaplewski 2013, 2015). These papers propose the application of the Danish weight function in the process of robust adjustment of navigational observations, and, in consequence, in determination of reliable coordinates of the vessel position.

This paper is an extension of the research presented in (Świerczyński, Czaplewski 2013, 2015). Our study aims at determining whether the application of different weight functions and different variants of parameters, which control robust observation adjustment, has a significant effect on the accuracy of vessel positioning in navigation networks. Numerical tests have been conducted on simulated observations, with three variants of the numbers of supernumerary observations in a measurement grid.

Robust M-estimation

Estimation of the vector of coordinates of a vessel $\mathbf{X} \in \Re^r$ can be based on the well-known linear form of the Gauss-Markov functional model:

$$\mathbf{v} = \mathbf{A}d\mathbf{X} + \mathbf{y}^{\circ} - \mathbf{y}^{obs} = \mathbf{A}d\mathbf{X} + \mathbf{L}, \tag{1}$$

in which $\mathbf{y}^{obs} \in \Re^n$ is an observational vector, $\mathbf{y}^{\circ} \in \Re^n$ is a vector of approximate values of observed geometric elements of a measurement network, $\mathbf{A} \in \Re^{n,r}$ is a matrix of known coefficients, $d\mathbf{X} \in \Re^r$ is such a vector of increments of approximate coordinates $\mathbf{X}^{\circ} \in \Re^r$ that $\mathbf{X} = \mathbf{X}^{\circ} + d\mathbf{X}$, and $\mathbf{v} \in \Re^n$ is a vector of theoretical observational corrections, such that $\mathbf{v} = -\boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon} \in \Re^n$ is a random vector of measurement errors. The following matrix of co-variances of measurement results corresponds with the functional model (1):

$$\mathbf{C_y} = \sigma_0^2 \mathbf{P}^{-1}, \tag{2}$$

where σ_0^2 denotes a global estimator of the coefficient of variance and $\mathbf{P} \in \Re^{n,n}$ is the matrix of observation weights. Let us assume that the observations under consideration are mutually independent. The matrix of covariance of the measurement results will then be the following diagonal matrix $\mathbf{P} = Diag\left(p_1, p_2, ..., p_n\right) = Diag\left(m_{y_1}^{-2}, m_{y_2}^{-2}, ..., m_{y_n}^{-2}\right)$, where m_{y_i} is a mean error of $i-$th observation.

Methods which are included in a broad family of M-estimations, involve determination of estimators $d\hat{\mathbf{X}}$, which minimise the arbitrarily-assumed objective function $\rho(\mathbf{y}; d\mathbf{X})$. Therefore, the parameters of model (1) are determined according to the following optimising criterion $\sum_{i=1}^{n} \rho(y_i; d\mathbf{X}) = \sum_{i=1}^{n} \rho(v_i) = \min$ (see e.g. Huber 1981; Hampel *et al.* 1986). Hence, when the gradient of the objective function $\rho(\mathbf{y}; d\mathbf{X})$ is determined, the system of normal equations in M-estimation can be noted as

$$\mathbf{A}^T \mathbf{w}(\mathbf{v})\mathbf{v} = \mathbf{A}^T \mathbf{w}(\mathbf{v})(\mathbf{A}d\mathbf{X} + \mathbf{L}) = \mathbf{0}, \tag{3}$$

where $\mathbf{w}(\mathbf{v})\mathbf{v} = Diag\left(w(v_1),...,w(v_n)\right)$ is a diagonal matrix of weight functions. M-estimator, which solves the following system of normal equations, is usually determined by the following iterative process:

$$d\mathbf{X}^j = -[\mathbf{A}^T\mathbf{w}(\mathbf{v}^{j-1})\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{w}(\mathbf{v}^{j-1})\mathbf{L}\,. \tag{4}$$

It should be noted that there is a different M-estimator for any weight function. For example, if one assumes that a matrix of weight functions has the following form $\mathbf{w}(\mathbf{v}) = \mathbf{P}$, then the M-estimator determined is, at the same time, an estimator of the least squares method minimising the objective function $\rho(\mathbf{y}; d\mathbf{X}) = \sum_{i=1}^{n} p_i v_i^2$ i.e.,

$$d\hat{\mathbf{X}}_{LS} = -[\mathbf{A}^T\mathbf{w}(\mathbf{v})\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{w}(\mathbf{v})\mathbf{L} = -[\mathbf{A}^T\mathbf{P}\mathbf{A}]^{-1}\mathbf{A}^T\mathbf{P}\mathbf{L}\,. \tag{5}$$

It should also be noted that the values of these weight functions are, in fact, modifications of the original weights of observations in the LS computational algorithm for estimators. Therefore, estimators in the least squares method can be used as values which trigger the iterative process of determining robust M-estimators.

A weight function plays a particularly significant role in the context of robustness. Assuming that an adjustment problem will be solved in an iterative process, then in each iteration the weight function defines the way in which original weights of observations y_i will be modified. A new weight of observation is determined in each iteration step in relation to the absolute value of the estimator $\hat{v}_i$. Individual M-estimators differ in their approach to determination of modified weights of observations. This paper considers four variants of weight functions of robust M-estimation (see e.g. Krarup, Kubik 1983; Labant *et al.* 2011; Ge *et al.* 2013; Nowel 2016):

1) Danish method

$$w(v_i) = \begin{cases} p_i & for \quad |\overline{v}_i| \le t \\ \exp(-l|\overline{v}_i - t|^g)p_i & for \quad |\overline{v}_i| > t \end{cases} \tag{6}$$

2) Huber's method

$$w(v_i) = \begin{cases} p_i & for \quad |\overline{v}_i| \le t \\ \dfrac{t}{|\overline{v}_i|}p_i & for \quad |\overline{v}_i| > t \end{cases} \tag{7}$$

3) L1 norm

$$w(v_i) = \frac{1}{|\overline{v}_i|}p_i \tag{8}$$

4) German-McClure method

$$w(v_i) = \frac{1}{\left(1+\overline{v}_i^2\right)^2}p_i\,. \tag{9}$$

It is noteworthy that the final form of the Danish weight function depends on the value of l and g. These quantities can be interpreted as parameters controlling the process of robust estimation. It must be emphasised that, in general (as well as in this paper), it is assumed that $g = 2$. The t is another controlling parameter which is present both in the Danish weight function and in Huber's method. This parameter determines the permissible interval $\Delta_{\overline{v}_i} = < -t; t >$ for standardised observational corrections. If a standardised correction $\overline{v}_i$ is outside the interval $\Delta_{\overline{v}_i}$ then the influence of this observation on the vector $\hat{\mathbf{X}}$ is limited accordingly by decreasing the weight of the $i-$th observation. If $\overline{v}_i$ does not exceed the permissible interval, then the weight of the observation is not modified. Selection of a specific value of the parameter t is related to the adopted level of probability. It is noteworthy that different assumptions are made for the other two weight functions (8) and (9). The original observation weights are modified for all measurement results, and the equivalent weight depends on the value of $\overline{v}_i$. The procedure for determining equivalent weights of measurement results in navigational measurement structures has been described in detail elsewhere (Czaplewski 2004; Czaplewski, Wiśniewski 2008; Świerczyński, Czaplewski 2015).

Experiment

The example considered in this paper refers to a situation described in (Świerczyński, Czaplewski 2015). The analysis presented here concerns a measurement network, in which navigational observations are acquired from coastal radar stations operating within the "VTS Gdańsk Bay". The system structure enables determination of a ship's bearing from five radar stations simultaneously, making up a measurement network, as presented in Figure 1.

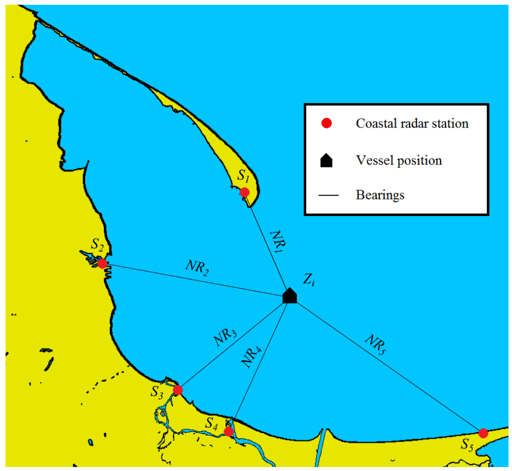

Fig. 1. A measurement structure using coastal radar stations

Coordinates of five coastal radar stations in the PL-UTM system are presented in Table 1.

Table 1. Coordinates of coastal radar stations in the PL-UTM system

Coastal station	Marking on Fig. 1	$X[m]$	$Y[m]$
Hel Lighthouse	S_1	6052476.63	357945.55
Gdynia_KPHarbour Master	S_2	6045669.81	341309.47
Gdynia_SHarbour Master	S_3	6045119.44	342083.22
Gdańsk North Port Harbour Master	S_4	6031298.79	348189.74
Radar Tower GórkiZachodnie	S_5	6027017.31	355714.79

Empirical analyses were conducted by simulating the position of a vessel $\mathbf{Z}_i = \left[X_{Z_i}, Y_{Z_i}\right]^T =$ $\left[6042470.00 \quad 348330.00\right]^T_{[m]}$. Theoretical coordinates of the position of a vessel were taken as approximate coordinates of the position of a vessel $\mathbf{X}^\circ = \mathbf{Z}_i$. In consequence, coordinates of the coastal station and the theoretical position of a vessel were used to simulate the results of measurements in the navigational measurement structure. The observations were simulated by contamination of the theoretical values of bearings with Gaussian errors with the expected value of $E(\varepsilon) = 0^\circ$ and the standard deviation of $\sigma = 0.5^\circ$. The values of simulated radar observations are shown in Table 2.

Table 2. Values of simulated radar observations

Vessel position	Bearings from coastal radar stations				
	NR_1 of S_1	NR_2 of S_2	NR_3 of S_3	NR_4 of S_4	NR_5 of S_5
Z	224.5°	114.4°	112.5°	0.1°	334.6°

Numerical computations were made for vector $\mathbf{y}$ in which there are no outlying observations as well as for a situation when one of the observations has a gross error. It was assumed for the experiment's sake that bearings determined from the Gdynia_KP Harbour Master were contaminated with a gross error of $e = 8^\circ$. The coordinates of the vessel's position were determined by the least squares method and by the method of robust M-estimation with the use of various weight functions i.e. Danish (6), Huber's (7), L1 norm (8) and German-McClure (9). The permissible standardised adjustment interval in the Dutch method and Huber's method was determined for the parameter $t = 2.5$. Two cases of the Dutch weight function were considered in the calculations; they will "dampen" the observation weights in a soft way – for $l = 0.01$ and a hard way – for $l = 1/k$. For the calculations, the mean error of a bearing is equal to the adopted standard deviation $m_{y_i} = 0.5^\circ$. The m_{y_i} was taken as the base for determination of a specific form of weigh matrix, which was used to determine LS and M_{split} of estimators and also initiated determination of a robust M-estimator.

It is commonly known that the detection effectiveness of outlying observations depends on the measurement of the reliability of a measurement network (see e.g. Prószyński 1994) and on the breakpoint of an M-estimator (see e.g. Kwaśniak 2011). For navigational networks whose geometric structure is similar to that considered in this paper (Fig. 1), the number of redundant observations is the main factor which affects these aspects. Therefore, three variants of observational vectors $\mathbf{y}^{obs}$, with different numbers of redundant observations, were developed on the basis of the simulated observations listed in Table 2. *Variant I* concerns a situation when vector $\mathbf{y}^{obs}$ contains radar observations from all the coastal stations, i.e. $\mathbf{y}^{obs} = \begin{bmatrix} NR_1 & NR_2 & NR_3 & NR_4 & NR_5 \end{bmatrix}$. Subsequent variants take into account one and two supernumerary observations, in *variant II* $\mathbf{y}^{obs} = \begin{bmatrix} NR_1 & NR_2 & NR_3 & NR_4 \end{bmatrix}$ and in *variant III* $\mathbf{y}^{obs} = \begin{bmatrix} NR_1 & NR_2 & NR_3 \end{bmatrix}$, respectively. The mathematical formulae presented in the previous chapter were used as the basis for determination of vessel coordinate estimators for three variants of sets of observations $\mathbf{y}^{obs}$, whose ultimate results are presented in Table 3.

Table 3. Results of increment estimations for parameter vectors of approximate values of $\mathbf{X}^\circ$ in a bearing grid

Method	Variant I		Variant II		Variant III	
	$dX[m]$	$dY[m]$	$dX[m]$	$dY[m]$	$dX[m]$	$dY[m]$
Observation set not contaminated by a gross error						
LS	93.27	–103.04	95.44	–121.87	96.65	–125.52
Danish (soft one)	93.27	–103.04	95.44	–121.87	96.65	–125.52
Danish (hard one)	93.27	–103.04	95.44	–121.87	96.65	–125.52
Huber	93.27	–103.04	95.44	–121.87	96.65	–125.52
L1 Norm	99.97	–119.52	99.97	–119.52	96.65	–125.52
German-McClure	105.95	–116.58	99.36	–119.80	96.65	–125.52
Observation set contaminated by a gross error						
LS	–333.81	–207.07	–325.34	–280.74	–242.75	–528.23
Danish (soft one)	44.37	–113.35	–6.78	–155.66	–242.75	–528.23
Danish (hard one)	163.55	–88.75	99.97	–119.52	–242.75	–528.23
Huber	69.34	–108.86	16.23	–147.61	–242.75	–528.23
L1 Norm	110.09	–117.28	100.54	–119.26	–242.75	–528.23
German-McClure	130.11	–115.29	90.81	–124.43	–242.75	–528.23

The LS estimators for sets of observations undisturbed by an outlying observation can be regarded as a correct position of a vessel and it can be a reference point for the remainder of the determined estimators. If vector $\mathbf{y}$ does not contain any outlying observations, the Danish and Huber's estimators are identical with the results of the least squares method. This is a consequence of the fact that none of the observations were qualified as outlying. It should be noted that for a set of observations undisturbed by an outlying observation, the estimators of the L1 norm and German-McLure differ slightly from the results of the least squares method. This is because these methods lack a specific confidence interval for standardised adjustments and the weights of all observations are modified in the iterative process. The results show clearly that contamination of one observation with a gross error has a negative effect on the position of a vessel as determined. The results of variant I and II show that the application of robust methods of estimation considerably reduced the effect of the gross error on the determined vector $d\mathbf{X}$. However, the results of robust estimation for variant III do not provide credible information on the position of a vessel. This is a consequence of taking into account insufficient supernumerary observations, which prevented detection of a gross error.

The results obtained in variants I and II are presented graphically in Figure 2. The LS estimate in Figure 2 is the reference point for the other results and it concerns the position of a vessel determined with an assumption that the vector of observation did not have a gross error.

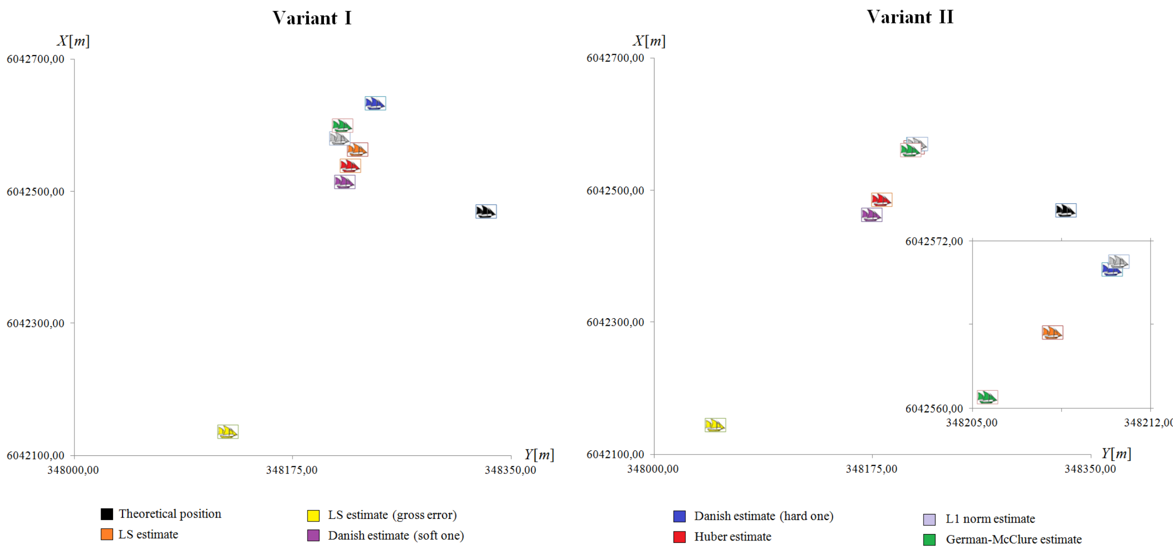

Fig. 2. Position of a measurement object determined by neutral and robust methods of estimation

The results of variants I and II show that the application of various weight functions in formula (4) results in different coordinates of a vessel's position. However, it must be emphasised that each of the methods under consideration is effective and it provides credible information on the position of the vessel. It is noteworthy that in both variants, estimators of the L1 norm and the German-McClure method demonstrate greater "convergence" with the position determined by the LS method for a "pure" set of observations.

Conclusions

The study presented in this paper aimed at improving the safety level for navigation in basins supervised by VTS systems. The authors propose that the credibility of the vessel coordinates determined could be improved by applying robust estimation, which is well-known in geodetic issues. This paper considers the application of various weight functions in the process of robust estimation of a vessel's position. The results of the numerical example show that methods of robust M-estimation can be applied in the adjustment of bearing networks. This applies especially when observations can have a gross measurement error. Empirical analyses have shown that the credibility of the vessel position is not affected by a choice of methods of robust estimation, but it is by the grid reliability. However, certain methods of estimation require that certain parameters controlling the process of robust observation adjustment should be adopted *a priori*. For this reason, it is "safe" to consider the application of the L1 norm and German-McClure weight functions.

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