

Determination of the Longitudinal Dispersion Coefficient in Lowland Streams with Occurrence of Dead Zones

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Abstract. Paper describes the field tracer experiments – determination of the coefficient of longitudinal dispersion in streams of lowland type (channels, small slopes and velocities) with expected occurrence of dead zones. Tracer experiments were carried out on three different streams in south Slovakia. The evaluation of field measurements confirmed in all cases the presence of dead zones, i.e. stream sections and regions with the appearance of small velocities that were formed due to the extensive presence of vegetation in the stream. These areas capture part of the transported substance (tracer) and then gradually release the substance and incorporated it back into the stream, creating a significant distortion of the tracer concentration time course. Strong influence of the dead zones raises the question of the adequacy using standard analytical solutions, whether for determining the coefficient of longitudinal dispersion or for modelling the dispersion of pollution or other substances carried by the stream.

Keywords: hydrodynamic dispersion, longitudinal dispersion coefficient, tracer field experiment, tracer, colouring agent.

Conference topic: Water engineering.

Introduction

The present legislation evaluating quality of water bodies (WF) in Slovakia is based on implementation of the Water Framework Directive (2000/60/ES) (European Commission... 2000). Concerning the Directive, it is required ecomorphological monitoring of WF, which is based on evaluation of the rate of anthropogenic impact. It does not refer only to stream bed, but also the state of environment nearby to stream is taking into consideration.

Dispersion coefficients are the crucial input parameters of transport processes models (Kosorin 1995; McInstyre *et al.* 2005; Řiha *et al.* 2002). Aim of this contribution is description of fact how the values of longitudinal dispersion coefficients, used as a one-dimensional model input data, affect the results of water quality numerical simulation.

The term hydrodynamic dispersion is, in hydrodynamic sense, the dispersion of the solute, which occurs in the flowing fluid. The dispersion of the substances together with the advection in the flow are underlying mechanism of dissolved particles movement in an aqueous medium. These phenomena help reduce the maximum concentration values of the solute in the flow. Solute spreads gradual to the sides due to the pulsation rate and different concentrations of the substance. The main characteristics of the dispersion are the dispersion coefficients of in the corresponding direction. The specification of these dispersion characteristics, plays therefore a key role in tasks solving the transport of pollutants in streams and in modelling water quality.

Theoretical background

Description of dispersion process in surface stream is based on one-dimensional advection-diffusion equation. It is the simplest mathematical formulation of this process from hydrodynamic point of view and it its form is:

$$\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} = D_x \left(\frac{\partial^2 C}{\partial x^2} \right) + S_x, \quad (1)$$

where: t is the time [s], C is the concentration of a substance [$\text{kg}\cdot\text{m}^{-3}$], D_x is the dispersion coefficient in the longitudinal direction [$\text{m}^2 \text{s}^{-1}$], v_x is the velocity of water flow in a given direction of flow [$\text{m}\cdot\text{s}^{-1}$], S_x is a function representing the sources of pollution [$\text{kg}\cdot\text{m}^{-3} \text{s}^{-1}$], x is the spatial coordinate – distance [m].

This equation includes two basic transport mechanisms: advection caused by water flow and dispersion caused by mass concentration gradient. Advection – diffusion equation (1) is based on assumption, that a transported substance is homogeneously distributed over the cross section and that the Fick's law of diffusion is applied, i.e. dispersion transport is proportional to the concentration gradient.

Using one-dimensional dispersion equation encountered in conditions of real streams to certain restrictions. Among them, we consider as the most significant restriction the impact of the dead zones.

Dead zones are parts of the streams with occurrence of secondary currents and zones with the appearance of small (or zero or even negative) velocities. These can be found on and along the banks of the stream, where in terms of flow the transported substance is collected and isolated from the main stream portion. Then this substance is gradually released and incorporated back into the main stream. In the streams, these zones are created by bigger obstacles, riparian vegetation, refuse the trunk and fallen tree branches, as well as guiding structures, oxbows, depressions and building objects. By (De Smedt *et al.* 2005) in the case of low speed flow, through gravel or permeable bottom (hyporheic zone) and the stream banks may flow considerable part of the flow of the stream. Influence scheme of dead zones by (Chanson 2004) is presented on Fig. 1.

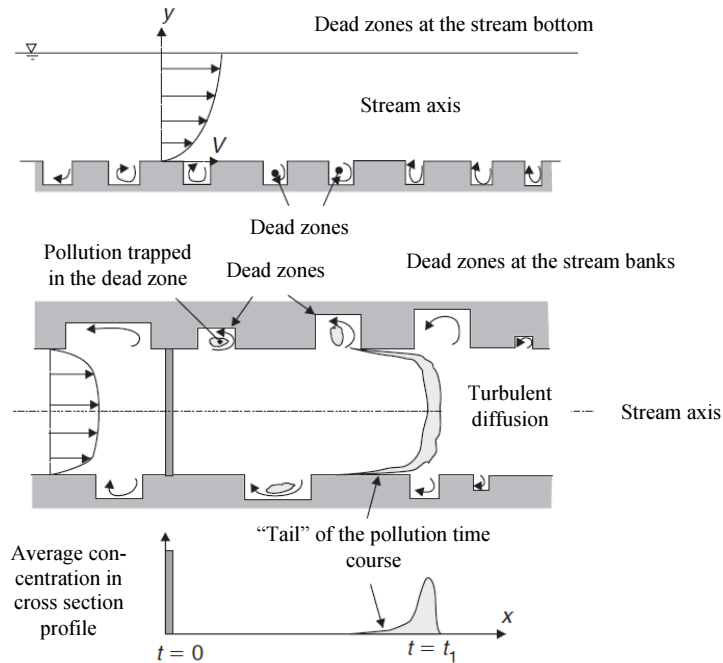


Fig. 1. Schematic drawing of the streambed with a “dead zones” (Chanson 2004)

The effect of these zones has been also described in several studies (e.g. Runkel 1998; De Smedt *et al.* 2005). The occurrence of these zones with the temporary accumulation of the transported substance commonly causes deformation of the distribution curve of concentration by the “pockets” formed by irregular boundaries of the stream. The substance is released later and more slowly, giving rise to the concentration time course curve the rapid onset, followed by “tail” (Fig. 1). Obviously, the number of streams with irregular geometry of the cross section along the stream the Fick's law cannot be applied even after a long period, since the concentration distribution due to large irregularities of the stream bed will never be a Gaussian (Nordin, Troutman 1980).

Analytical solution, assuming various simplifications of the (1) can be written in the form of one-dimensional equation (Fischer *et al.* 1979):

$$C(x,t) = \frac{M}{2A\sqrt{\pi D_x t}} \exp\left(-\frac{(x - v_x t)^2}{4 D_x t}\right), \quad (2)$$

where: $C(x,t)$ is a mass concentration [$\text{kg} \cdot \text{m}^{-3}$] in a place and time; D_L is the longitudinal dispersion coefficient [$\text{m}^2 \cdot \text{s}^{-1}$]; A is a discharge area in a stream cross-section [m^2], G is the mass of a tracer [kg], u is a mean velocity [$\text{m} \cdot \text{s}^{-1}$], x is a distance [m], t is time [s].

Interpreting the solution of the authors (Socolofsk, Jirka 2005) show that the analytical solution of (1), shown as (2), has a form of Gaussian normal distribution with parameters of normal distribution (e.g. standard deviation σ , etc.). In fact, this curve does not have the exact shape of a Gaussian normal distribution, but it has asymmetric form. This results from the fixed position of the observer on the bank (if the observer will move with the same speed as the water flow, the concentrations distribution from the observer's point of view will have symmetrical shape of the Gaussian normal distribution).

This solution is currently considered as the standard solution, but the validity of this solution is limited to flow without barriers, with the assumption of a symmetrical movement of the particles in the flow direction as well as the direction of flow. In real streams, however, this assumption is not true: The movement of particles is slow down (retarded) by the accumulation of particles in the so-called dead zones of the streams.

For pollution transport in streams with transient storage can be the (1) enlarged by the term, expressing the exchange between the transient storage (dead zones) and the main channel. If we can assume the water flow and transport parameters as constant, these equations can be written in the form (Nordin, Troutman 1980; De Smedt *et al.* 2005):

$$\frac{\partial C}{\partial t} = D_x \left(\frac{\partial^2 C}{\partial x^2} \right) - v_x \frac{\partial C}{\partial x} - \alpha (C - C_s) + S_s; \quad (3)$$

$$\beta \frac{\partial C_s}{\partial t} = \alpha (C - C_s), \quad (4)$$

where: the C_s is the concentration of the substance in the storage zone [$\text{g} \cdot \text{m}^{-3}$], α is the mass exchange coefficient between the main channel and the storage zone [s^{-1}], β is the ratio between the storage zone and the main channel cross-sectional area [-]. If α or β becomes zero, the (3) reduces to (1).

Field measurements

The tracer experiments were performed at the streams (channels) in south Slovakia – Malá Nitra (Location 1), Šúrsky channel (Location 2) and Malina (Location 3). All these locations are part of regional irrigation (drainage) systems. As stated in other works (Dulovičová, Velísková 2007, 2010) silting and subsequent aggradation of these streams is an ongoing process, affecting also the dead zones occurrence, but at all locations only small layers of silt were observed. From all tracer experiments, only the experiments with instantaneous tracer input and long distances were considered.

The examined section of stream Malá Nitra is situated within the village Veľký Kýr (N+48.179458°, E+18.155185°). The experiments described in this paper were performed in sections with lengths of 785, respectively 1340 m. In both cases, it was a straight section of the stream, without significant directional changes, but prior the second measurement profile slightly curve of the channel in both directions was located (left & right bend). In the examined section of the Malá Nitra stream the bottom width was about 4 m, height of the banks about 2.5 meters, bank slope is approximately 1:2. It should be mentioned, that the original prismatic cross section was not fully preserved and its form was modified as a result of ongoing morphological processes. The flow rate in the whole measurements period ranged from 0.138 to 0.553 $\text{m}^3 \cdot \text{s}^{-1}$, during the measurement for the model tests was the discharge quite stable, from 0.230 up to 0.235 $\text{m}^3 \cdot \text{s}^{-1}$. The hydraulic roughness coefficient was determined on the basis of field measurements and hydraulic calculations according to the method of (Limerinos 1970), and has a value of $n = 0.035$. Stream bed slope, determined by geodetic levelling, was found constant and was approximately 1.5 ‰. The shape of the channel can be considered in the examined section as a prismatic with a width at a water level of about 5.5 meters and a depth in the range of 0.4 to 0.6 meters.

In second case, the tracer experiments were performed on straight channel section of the Šúrsky channel, situated close to the village Svätý Jur (Slovakia, N+48.230350°, E+17.201481°). The field measurements were made in 300 up to 500 m long straight channel reach with relatively prismatic cross section profile. The channel bed was 4–5.5 m width at water level, max. depth was in the range 0.4–0.8 m, velocity 0.21–0.36 $\text{m} \cdot \text{s}^{-1}$ and a discharge was 0,38–0,43 $\text{m}^3 \cdot \text{s}^{-1}$. The range of determined longitudinal dispersion coefficient was 0.63–0.82 $\text{m}^2 \cdot \text{s}^{-1}$.

The tracer experiments in the third location were performed on the Malina channel, located in the cadastral areas of Lab and Zohor municipalities. (N+48.334771°, E+16.967445°). The experiments were carried out on selected stream section with a length of 1415 m. It was a straight section of the Malina channel, without significant directional changes. Originally constructed cross section shape was significantly influenced by vegetation. Measured flow during the experiments was 0.408 $\text{m}^3 \cdot \text{s}^{-1}$. The water level slope, specified by levelling measurements, was about 0.45‰. The channel shape in the examined channel section can be considered prismatic, the surface width was about 5 m, the average depth was 0.88 m (maximum depth was about 1 m), the determined dispersion coefficient was 0.95 $\text{m}^2 \cdot \text{s}^{-1}$.

During the measurements, we found on all streams deformation of the concentration time courses, which shows significant presence of dead zones. These were formed by the stream bed irregularities, as well by the vegetation along the stream banks and on the stream bed.

Examples of the field tracer experiments are shown on Fig. 2–4.

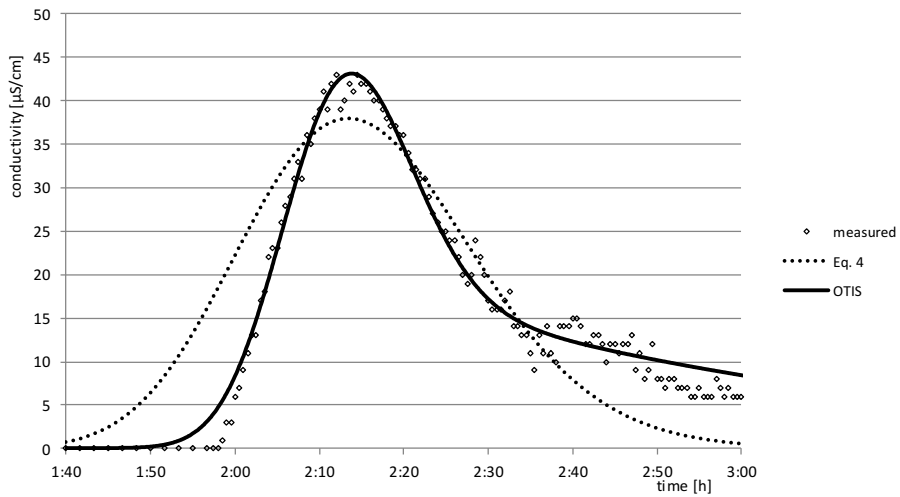


Fig. 2. Tracer experiment Nr. 5-V in location 1 – Malá Nitra: Time courses of the measured and modelled data, using (2) and OTIS-P

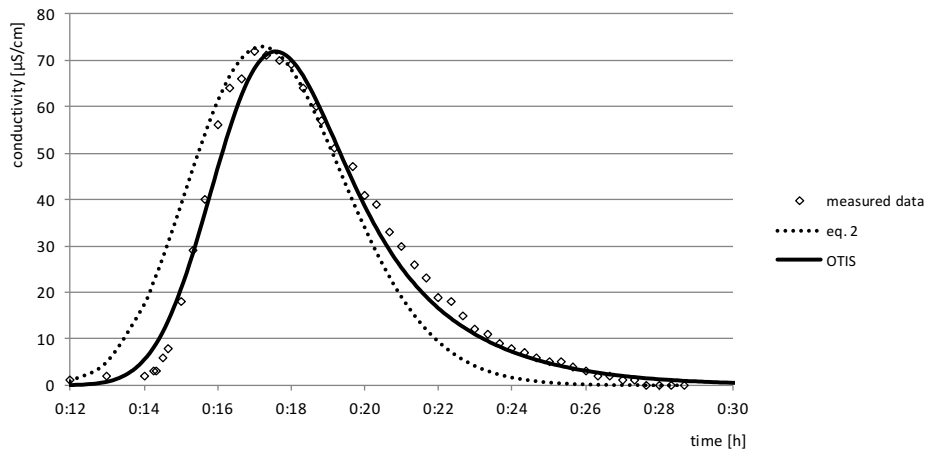


Fig. 3. Tracer experiment Nr. 7-II in location 2 – Jurský Šúr: Time courses of the measured and modelled data, using (2) and OTIS-P

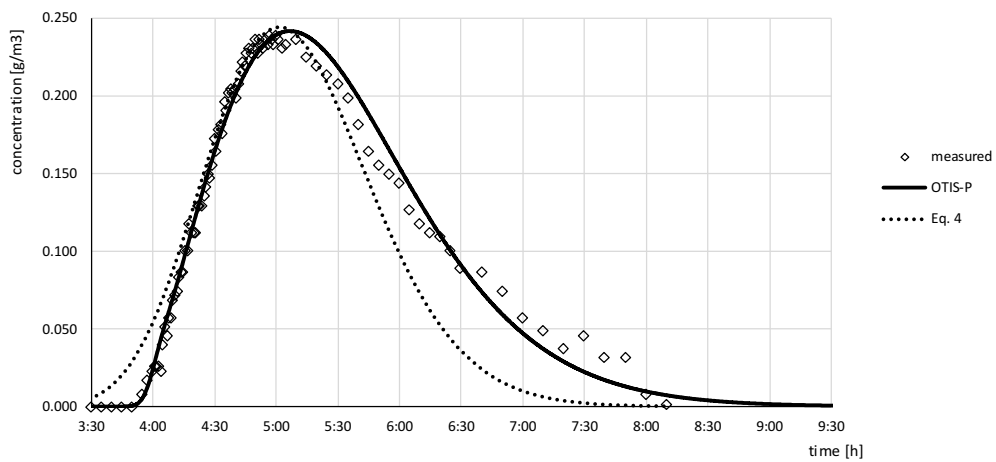


Fig. 4. Tracer experiment Nr. 3 in location 3 – Malina: Time courses of the measured and modelled data, using (2) and OTIS-P

Discussion

During the measurements, the presence of dead zones was detected based on the deformation of the concentration time course. The dead zones were established by channel irregularities and the vegetation on the banks of a channel.

As indicated in the paper, we consider the existence of “dead zones” in the streams as a fundamental problem when determining the longitudinal dispersion coefficient. This phenomenon should be taken into account in modelling, respectively. in the dispersion coefficients values determination methods. In the figures (Fig. 2–4) the measured concentration time course curves are indicated, as well as the time courses of the modelled concentration curves, which are computed on the assumption of a Gaussian distribution, resulting from (2). As shown in the figure, between the measured and modelled values is a weak conformity.

Statistical analysis of the modelled data shown that in large number of the experiments performed, the measured values do not have the character of the Gaussian normal distribution, or more precisely the character of the (2). The statistical analysis has proved, that the statistical test of the hypothesis H_0 (the measured distribution function is a distribution of the theoretical distribution), using a high confidence level ($\alpha = 0,95$) has been confirmed for the model, based on (2) in 4 cases only (less than 20% of total performed measurements).

For this reason, we consider the use of the analytical solutions for dispersion modelling problematic and limited to the streams, where the influence of dead zones in the dispersion process can be neglected. As an alternative to use of the analytical solutions may be a simplified (one-dimensional) mathematical models considered (Runkel 1998; De Smedt *et al.* 2005). Such model can comprise the impact of dead zones in the stream. Also, these models, however, require input of the dead zones parameters (e.g. cross sectional area ratio between dead /mainstream zone, transfer coefficients etc.). These coefficients can also be determined in several ways, but the most reliable way is to perform the tracer experiment and to determine required parameters based on the measurements.

For the demonstration and evaluation purposes we perform the analysis of three field tracer experiment, one from each location. In each of the tracer experiment we perform the determination of the longitudinal dispersion using three methods. The first method was based on comparison of the analytical solution of the dispersion equation derived by using simplified assumptions (2) with the measured data.

The second method of the dispersion coefficient determination is based on statistical data evaluation. It uses the measured concentration time course (assumed Gauss distribution) and set up the time difference ranging from $-\sigma$ to $+\sigma$, ie. approximately from the 13.6 and 86.4 percent quantile of measured values. The values of the dispersion coefficient then were determined on the basis of the following equation (Socolofsky, Jirka 2005).

$$D_x = \frac{\bar{u}^2 \sigma^2}{2t_v}, \quad (5)$$

where: σ is the standard deviation of the measured concentration curve [–], \bar{u} is the average flow velocity [$\text{m}\cdot\text{s}^{-1}$], t_v is the time for the peak (maximum) of the tracer concentration time course (the time at which the maximum tracer concentration is achieved).

The last – third method is base un use of the model OTIS, eventually the OTIS-P model (Runkel 1998). OTIS is a mathematical simulation model used to characterize the fate and transport of water-borne solutes in streams and streams. The governing equation underlying the model is the advection-dispersion equation with additional terms to account for transient storage, lateral inflow, first-order decay, and sorption. This equation and the associated equations describing transient storage and sorption are solved using a Crank-Nicolson finite-difference solution. OTIS-P, a modified version of OTIS, couples the solution of the governing equation with a nonlinear regression package. OTIS-P determines an optimal set of parameter estimates that minimize the squared differences between the simulated and observed concentrations, thereby automating the parameter estimation process. (Runkel 1998)

Results of the longitudinal dispersion coefficient determination are summarized in Table 1.

Table 1. Values of the longitudinal dispersion coefficient, determined by various methods

Location (stream)	Method 1	Method 2	Dx - method 3		
	fit to the (2)	Statistical evaluation	OTIS-P		
	Dx [$\text{m}^2 \text{s}^{-1}$]	Dx [$\text{m}^2 \text{s}^{-1}$]	Dx [$\text{m}^2 \text{s}^{-1}$]	α [s^{-1}]	β [–]
Loc. 1 Malá Nitra (exp. Nr. 5-V)	1.17	1.377	0.3637	$1.18 \cdot 10^{-4}$	0.225
Loc. 2 Jurský Šúr (exp. Nr. 7-II)	0.64	0.766	0.3737	$4.08 \cdot 10^{-4}$	0.078
Loc. 3 Malina (exp. Nr. 3)	0.95	0.607	0.0692	$3.55 \cdot 10^{-4}$	0.469

Explanation of the results from the Table 1: there are various values of the longitudinal dispersion coefficient, depending on the local hydraulic conditions. Results of the OTIS-P are graphically shown on the Fig. 2–4. Field experiments in the location 2 (Jurský Šúr) were performed in the early spring (March), so the vegetation in the stream was nearly not present, because of this is the parameter β very small. On the other hand, in the other two locations were the experiments performed in July and August, so the vegetation was present, especially in the third location (Malina). In this location was the vegetation not only close to the stream banks, but also in the main channel – this explains the extremely high value of the parameter β – almost the half of the stream cross section can be regarded as the transient storage zone (dead zone) – see Fig. 5.



Fig. 5. Water vegetation in Malina stream

Conclusions

In this paper, we present the results of the field tracer experiments, performed on three streams (channels) in south Slovakia. In all streams the presence of transient storage (dead) zones was confirmed, which was caused mainly by the various extent of vegetation presence in streams.

The presence of dead zones causes distortion of the tracer concentration time courses, which results in problems in longitudinal dispersion coefficient determination. Values of the longitudinal dispersion coefficient, determined by various methods are presented in Table 1. As can be seen in this table, the differences in the determined longitudinal dispersion coefficients are significant, especially when the values were determined by the regression package OTIS-P. In this case, not only the longitudinal dispersion coefficient, but a set of three parameters are used in this model.

This means, that in relation to the general effort to the closest possible conformity between measured and modelled concentration time courses, the method of determining the coefficient of dispersion has to be carefully selected. We believe that the determination method of this key parameter should be in accordance with the method of use of a particular model or method to achieve maximum conformity, e.g. (Halaj *et al.* 2013; Szomorová, Halaj 2015). Such an approach would require in most cases to perform tracer experiment and determination of the dispersion coefficient with regards of the future model use.

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Contribution

The first author – M. Sokáč declare involvement in conception and design of the work, data acquisition, analysis and data interpretation.

Disclosure statement

The author declare that they do not have any competing financial, professional, or personal interests from other parties.

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