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The Use of Gravity Model in Spatial Planning

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Abstract. The GIS tools facilitate organised and formal creation of models presenting both the current state of and the forecasted changes in physical environment (ones that will occur if specific conditions are satisfied). Formulated in 1962, D. L. Huff's algorithm is gaining in popularity nowadays; it helps determine the probability of a particular venue being chosen from amongst several competitive ones. Initially, it was used to define the optimal location of new points of sale; but with the technological progress of the GIS and with an increased demand for studies on distribution of venues or service points, its scope of use in urban planning is becoming more and more extensive. The results of the study support decision-making processes and are invaluable help in selection of optimal locations.

This article presents how Huff's algorithm and Lakshmanan-Hansen's channel pass-through model can be used in physical planning of cities with the focus on new trade centres. The research was based on the case study of Warsaw – the biggest city in Poland. The analysis conducted showed not only what market share particular centres had, but it also demonstrated what changes could be expected upon appearance of new trade centres.

Keywords: GIS, spatial planning, Huff model, market share.

Conference topic: Sustainable urban development.

Introduction

Spatial planning is one of the domains which is, both theoretically and practically, characterised by being largely interdisciplinary. When planning the allotment of individual areas one should aim at cross-compliance, complementariness and cohesive functioning of the spatial objects planned (Pooler 1987; Fotheringham 1983). Therefore, it appears natural that Spatial Information Systems are used for these purposes. The systems help their users organize and formally produce models which present both the current situation and the forecast for future changes.

Searching for spatial location for various activities is the main goal of spatial planning (Cartwright 1973). The location of a specific planned function depends on numerous diversified factors, conditions and limitations (Alterman, Page 1973). The location of a market activity depends on environmental conditions, social needs, economic potential and technical feasibility.

The GIS tools are applicable everywhere where presentation of a certain issue in the spatial dimension is necessary. Planning and spatial management are areas where the GIS offers extensive opportunities for application. Currently, in Poland, the GIS technologies are used first of all in the process of planning documentation development, mainly as a tool for spatial data gathering and presentation (Kurowska *et al.* 2014). Development of the GIS technologies caused that recently the system has been supporting decision-making processes in planning (MacEachren *et al.* 2005).

The objective of the study is to demonstrate the possibilities of using Huff's model and Lakshmanan and Hansen retail gravity model to plan new investments in the urban space. The study also aims to designate coverage for existing stores and the assessment of the changes that will occur in the urban space (the coverage of the shopping centers) after the appearance of the Gallery Białołęka and the Gallery Wilanow in Warsaw.

Using Huff's model and Lakshmanan and Hansen retail gravity model authors try to determine whether the emergence of new centers will adversely affect the market share of all gallery or just those located in the immediate vicinity, and what change can we expect in relation to this process.

The object of the study is 17 major commercial buildings in the Warsaw area, which were selected based on questionnaire surveys. Based on their responses attractiveness of shopping centers were determined. The model is built on a single city level (the distance between the centers are small), it was decided to use demographic data knotted directly to the point address. The basis to determine the distance is a detailed road network for the agglomeration of Warsaw.

Role and availability of data

One of the basic elements of the Spatial Information System is represented by the database with (spatial and descriptive) information on real-world objects. According to the Lexicon of Geoinformatics, "data" is to be interpreted in the following terms: presentation of information which is suitable for communication, interpretation and processing. Data exist in form of signs (including digits and letters), symbols, pictures, and other recorded forms which are comprehensible to a user or which can be processed. Data on their own do not carry any meaning – it is only in the process of user interpretation that they become information which can be used for knowledge development (Gaździcki 2001).

A more and more extensive use of the GIS in a multitude of disciplines often forces users to search data relevant to one's scope of interests and, at the same time, suitable for implementation in analytical procedures. An increased public attraction to spatial data of various kinds has led to emergence of companies specialising in provision of GIS-compatible spatial data. The only challenge that a user has to face is the selection of a data provider and data which – from the perspective of the project undertaken – will have the best parameters. The issues which should be considered include: project profitability, data accuracy and validity, contents, spatial structure and attribute tables, frequency and scope of updating, licensing methods, and format.

The majority of data stored in the database have spatial reference. No matter if the collected data are linked with a particular address, street, post code or commune, they can be transferred onto a map. The most popular form of storage of information on the spatial location of an object is based on the address and the process involving conversion of address data into geographical coordinates and object mapping is called geo-coding.

GIS users also take advantage of demographic and economic data – a wealth of knowledge on human potential in a particular area. Deriving from various research centres, the data regard customers' demography, affluence or shopping habits and they are compiled and handled by commercial companies, then they are made available as a finished product ready for use in analyses.

The commercial databases available in the Polish market are integrated sources of statistical data on the number, structure and affluence of inhabitants of a given area. The data are prepared for a single building (address point) or for any specified statistical area. Among the most important demographic variables there are:

– composition of the population broken down by gender and age (with age brackets);
– number of households and inhabitants;
– purchase power of inhabitants (PPI) and purchase power in retail;
– structure and kind of buildings.

However, besides the data acquired from external sources, which have huge analytical potential, there are also internal data of different institutions imported to the system. The bigger the volume of data we have at our disposal, the more advanced and conclusion-abounding our analyses can be.

A database implemented to the system is its most important element. It affects system's functions and scope of use. Without correct data (with proper specificity and accuracy) even the best-trained team equipped with a specialist software will not be able to provide an elaborate answer to user's question. The right selection of object classes at the design phase of a particular model of reality is essential to retain system's core scope of information and has a fundamental influence on the efficiency of data management in the future (Felcenloben 2011).

Use of huff model in urban space planning

Assuming that the GIS database is a model of reality transferred to the analytical system, the GIS environment enables modelling, simulation and future optimisation of processes happening in the world around us. Simulations performed on models with a high similitude to the reality include, for instance: analysis of spatial changes upon erection of a new object (shopping centre, school, hospital) or examination of the efficiency or capacity of traffic networks (under certain conditions).

A method of forecasting spatial transformations and examining the impact of particular centres on their neighbourhood which has enjoyed the biggest popularity recently is the gravity model (Chojnicki 1966; Fotheringham 1983). It is based on the so-called Reilly's Law (Reilly 1931), which says that two centres attract people inhabiting the area between them proportional to the number of people and inversely proportional to the square of the distance between the centre and inhabitant's location (Reilly 1929; Thrall 1988).

Gravity models have been expanded and improved with potential models and attraction models, among which Huff's model of relative gravity is the one used most frequently (Huff 1963, 1964). Initially employed in market analyses, it found its place also in urban and transport studies (Czyż 2002), to name just two domains. It is used to analyse the current situation and to determine the sphere of influence of a particular centre. When strategic investment decisions are made, such as; selection of a new location for the centre, Huff model makes it possible to simulate the effectiveness of options at hand (Dramowicz 2005).

With the assumption that there are m branch-specific centres in the area under analysis (Huff 1963):

$$C_j\{(x_j, y_j), F_j\}, \tag{1}$$

where: (x_j, y_j) – location coordinates of (j) centre in the area; F_j – attractiveness of (j) centre [0,1]; j = 1, 2, ..., m, the probability of the (j) centre being visited by the inhabitant of point (i) in the area is calculated according to the following formula (Huff 1963):

$$p_{ij} = \frac{\frac{F_j}{d_{ij}^{\lambda}}}{\sum_{j=1}^{m} \frac{F_j}{d_{ij}^{\lambda}}}, \tag{2}$$

where: d_{ij} – drive time measured between the place of residence and the centre; λ – exponent of the drive time; F_j – attractiveness of (j) centre <0,1>; pij – probability of (j) centre being visited by the inhabitant of point (i).

The correct estimation of the real drive time is difficult, requires a high-accuracy database and correctly configured algorithms to set a route matching a defined criterion. The time necessary to cover the distance also depends on the time of the day (particularly in highly urbanised areas), road conditions or traffic congestion. In order to exclude the miscalculations caused by the above-mentioned factors, simplify the model and arrive at the most precise Huff model, it has been generally accepted to use alternatively readings of the distance between client's address (i) and the centre (j).

Huff's algorithm has been extended into Lakshmanan-Hansen's channel pass-through model. In this kind of analysis the financial potential of local population is the limiting condition. This new variable allows for calculation of consumer's total expenditure in a particular trade centre (Lakshmanan, Hansen 1965).

$$W_{ij} = V_{ij} p_{ij} = W_j = V_{ij} \frac{\frac{F_j}{d_{ij}^{\lambda}}}{\sum_{j=1}^{m} \frac{F_j}{d_{ij}^{\lambda}}}, \tag{3}$$

where: W_{ij} – expected expenditures of consumers inhabiting point (i) while shopping in centre (j); V_i – total expenditures of consumers inhabiting point (i); d_{ij} – drive time measured between the place of residence and the trade centre; λ – exponent of the drive time; F_j – attractiveness of (j) centre [0,1];

The total sum of the results for every location, irrespective of trade centre, shows how big market share a particular trade centre has.

$$W_j = \sum_{i=1}^{n} V_{ij} \frac{\frac{F_j}{d_{ij}^{\lambda}}}{\sum_{j=1}^{m} \frac{F_j}{d_{ij}^{\lambda}}}, \tag{4}$$

where all variables are as in formula (3).

What is an important element of the analysis is border definition for the studied area. The algorithm is based on the assumption that the financial channel pass-through is contained within the borderlines of a particular zone; therefore, it is recommended to define its borders in such a way that there is minimal interaction with what is beyond its borders. For the sake of this analysis it was presumed that the studied area is not entirely hermetic, but the values of the in-flow and out-flow channels are mutually balanced. The analysis exclusively accounts for these address points which are within a distance of no more than 40 km from a trade centre.

The aim of the analysis is the construction of a model which would enable the assessment of a particular address point in terms of the probability of it being visited, on the one hand, and the establishment of consumers' expenditures in a specific trade centre, on the other hand, which is to be carried out by means of the matrix of financial flows between shopping zones and trade centres (Lakshmanan-Hansen's model). Upon the analysis it will also be possible to define each trade centre's market share and to examine its range of attractiveness and changes in its market share once a new trade centre appears (Galeria Północna, Światowida street; Galeria Wilanów, Przyczółkowa street). Huff model can be used in any analysis of a new point in the existing network. No matter if one wants to examine the area coverage of airports, number of prospective patients in hospitals, coverage of particular educational or shopping centres, one will find a properly prepared Huff model (with appropriate parameters) useful in predicting the transformations which will take place in the studied environment depending on the intended investment location and potential.

Taking into account the availability of data and model's practical usage, Huff model was designed on the basis of the shopping centres currently located in Warsaw. By means of Huff model any address point can be assessed with regard to the probability of it being visited and to changes in the attractiveness of each shopping centre upon emergence of new trade centres (Galeria Północna, Światowida street; Galeria Wilanów, Przyczółkowa street).

The aim of the model constructed is to examine the dependencies between particular centres in the micro-scale (within an agglomeration); therefore, it has been decided to use socio-economic data with the highest accuracy available with direct linkage to the address point data. Each point in the base has a detailed description of its address and the number of residents (in the building).

Upon the analysis of the instruments available in the market and their functionality, MapInfo PRO 12.5 software was selected and enhanced with the overlay programme – Vertical Mapper – which was to enable analyses based on raster data.

The most important element in the procedure of market coverage testing for trade centres is the correct establishment of their attractiveness. Huff model is exceptionally universal in this respect. Regardless of the characteristics of a particular centre, a properly constructed model should yield the attractiveness index with due consideration to other variables. In the traditional approach the following variables are proposed:

– total usable area of the venue;
– total trade area;
– number of parking places;
– accessibility (public transport);
– number of tenants.

In practice, analysts do not use this solution, though. The data on the surface area of a trade centre, the number of its shops or parking places are rarely easily accessible. Additionally, the attractiveness of a particular centre is highly dependent on other factors, such as: availability of leading brands, impression a centre makes on its customers, marketing campaigns, and many other factors which have influence on consumer's subjective feelings. It is believed that a model based on direct market research is more reliable and more commonly in use.

The market coverage of each centre was determined upon the tests made on selected shopping centres in Warsaw. Seventeen points were chosen, their address details were gathered and then they were geo-coded and displayed on the map (see Table 1).

Table 1. Location of trade centres in Warsaw

Name	Street	No.	City	X	Y
Blue City	Aleje Jerozolimskie	179	Warsaw	20.954942	52.212956
Carrefour Bemowo	Powstańców Śląskich	126	Warsaw	20.930259	52.264231
Arkadia	Jana Pawła II	82	Warsaw	20.983648	52.255793
Reduta	Aleje Jerozolimskie	148	Warsaw	20.951542	52.212432
Targówek	Głębocka	15	Warsaw	21.057329	52.303899
Wileńska	Targowa	72	Warsaw	21.035663	52.254724
Wola Park	Górczewska	124	Warsaw	20.931059	52.241248
Fort Wola	Połczyńska	4	Warsaw	20.927377	52.223293
Galeria Mokotów	Wołoska	12	Warsaw	21.003403	52.17882
King Cross Praga	Jubilerska	1	Warsaw	21.119769	52.235889
Klif	Okopowa	58/72	Warsaw	20.979127	52.246704
Promenada	Ostrobramska	75	Warsaw	21.105829	52.232379
Sadyba Best Mall	Powsińska	31	Warsaw	21.062221	52.187022
Złote Tarasy	Złota	59	Warsaw	21.002637	52.229564
Land	Wałbrzyska	11	Warsaw	21.025789	52.173194
KEN Center	Ciszewskiego	15	Warsaw	21.040476	52.153715
Plac Unii City Shopping	Puławska	2	Warsaw	21.020117	52.212351

For the purposes of this study the value of the index at issue was established upon the survey carried out on a group of 50 prospective clients of the analysed shopping centres. It was assumed that a potential client is a person who resides in Warsaw and is at the age of 20–40 years. The respondents were supposed to assess these trade centres by their attractiveness for shopping (attractive – 1, inspiring no interest – 0). Having collated the responses and averaging (mean value), the index values for Warsaw trade centres equal as shown below (see Table 2).

The surveys conducted made it also possible to select – out of 58 large commercial centres in Warsaw – those centres which had the attractiveness level at above 0.5; these centres were acknowledged as main competitors of the emerging shopping centres and were at the basis of the further study.

Table 2. Attractiveness index for trade centres in Warsaw and the share of the market before and after the appearance of new centers

Name	Attractiveness	Market share before [%]	Market share before [mln PLN]	Market loss* [%]
Blue City	0.9	5.71	4 611	8.47
Carrefour Bemowo	0.7	4.46	3 602	8.62
Arkadia	1	6.89	5 563	8.65
Reduta	0.8	4.73	3 814	8.59
Targówek	0.8	4.31	3 480	**10.57**
Wileńska	0.9	6.68	5 391	8.75
Wola Park	0.9	5.66	4 564	8.36
Fort Wola	0.6	3.65	2 949	8.49
Galeria Mokotów	1	6.47	5 219	**9.09**
King Cross Praga	0.8	4.76	3 842	**9.16**
Klif	0.7	4.96	4 000	8.30
Promenada	0.9	5.76	4 646	**9.10**
Sadyba Best Mall	0.8	4.91	3 966	**9.75**
Złote Tarasy	1	7.50	6 052	8.33
Land	0.6	4.08	3 290	**9.22**
KEN Center	0.6	3.88	3 128	**10.24**
Plac Unii City Shopping	1	7.51	6 065	8.51
Galeria Białołeka	1	3.70	2 987	
Galeria Wilanów	1	4.38	3 538	
TOTAL MARKET		100	**80 708**	

* loss of market share experienced by trade centres upon emergence of two new centres

What is the basis for Huff model construction is a point grid defined by the attractiveness index. Since none of the sources provides strictly what kind of point objects are to be used, the authors of this analysis took into consideration one of the following solutions available: analysis relying on the address points in the database or on the road junctions in the existing communication network.

Having reviewed the data in terms of:

– number of objects in the set (over 130,000 residential buildings and 50,000 road junctions);

– available instruments and database options for having the extra attribute of distance added;

– spatial structure and regularity of point distribution,

the authors decided to work on the road junctions.

By means of the available optimisation instruments, over 50,000 road junctions were described by the distance measured between the client's location (i) and the trade centre (j), the length of which was taken along the road axis to minimize the distance parameter.

Such a point grid constitutes the basis for Huff model construction. According to the formula (1, 2), each of the road junctions was matched with the value of the shopping probability in a particular trade centre and upon the use of a proper interpolation method Natural Neighbourhood. This method is most appropriate because data points are distributed with uneven density. It is unnecessary to specify parameters such as radius, number of neighbours or weights. Natural Neighbourhood has local minimum and maximum values in the point file and can be set to limit the overshoots of local high values and the undershoots of local low values.

Huff model of the market coverage of the seventeen shopping centres was created. With such a description of the points, zones were marked to indicate the strongest attraction (attractiveness) of a specific centre (see Fig. 1).

Huff models described here are often used for the discovery of gaps in a studied area. From the investor's point of view, the places with the lowest attractiveness index – where an existing centre fails to successfully attract the inhabitants – are highly interesting. Similarly, in the case of investment projects related to a new health-care centre, airport or school, the areas with the lowest attractiveness index have the poorest land development and should be considered for a potential location in the first place.

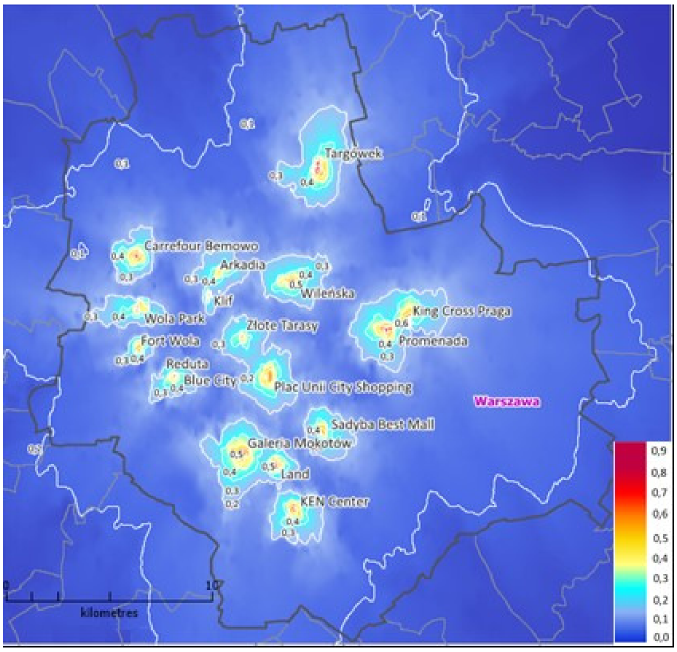

Fig. 1. Market coverage of shopping centres in Warsaw

Predictive modelling – change in market coverage

What is indispensable prior to the implementation phase of any investment is the examination of both the potential of the chosen location and its surroundings, on the one hand, and the changes which will take place in the market following the commissioning of the finished buildings, on the other hand.

At the moment there are two major investment projects under way in Warsaw: Galeria Wilanów (Przyczółkowa street) and Galeria Północna (Białołęka District). In order to demonstrate the practical options of the model for forecasting market changes, the two investment projects were added to the base of points defining the location of the existing trade centres and only then – with the enlarged set of data – was the whole analytical procedure performed.

The results were useful for the visualization of the predicted market coverage of each trade centre in Warsaw (see Fig. 2) and provided insight into the changes to take place in the market structure, customers' migration and their expenditures in particular trade centres (Table 2).

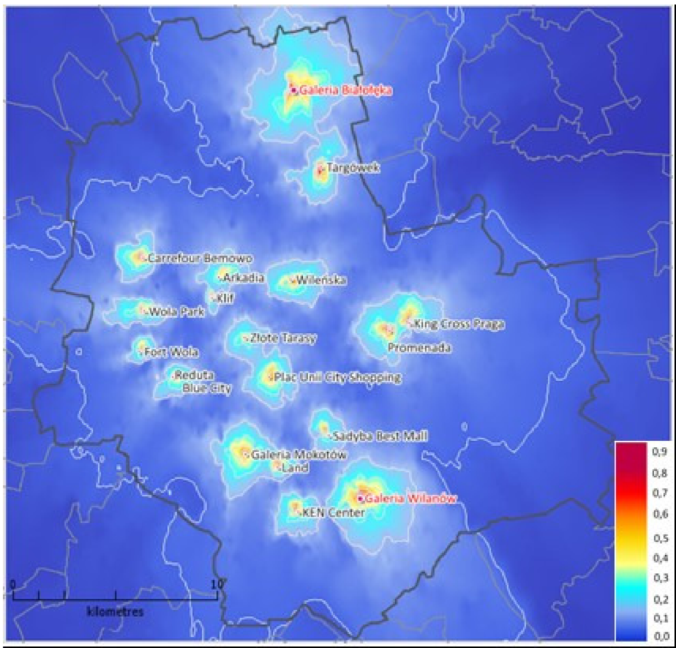

Fig. 2. Market coverage of shopping centres in Warsaw upon inclusion of two new centres: Galeria Wilanów and Galeria Białołęka

Upon the analysis of Huff model it can be concluded that a trade centre strongly affects the inhabitants of its immediate neighbourhood if the attractiveness index oscillates between 0.5–0.9. Any value from this range indicates that it is far from probable that these consumers will chose another trade centre. The further from a centre, the stronger the influence of adjacent centres.

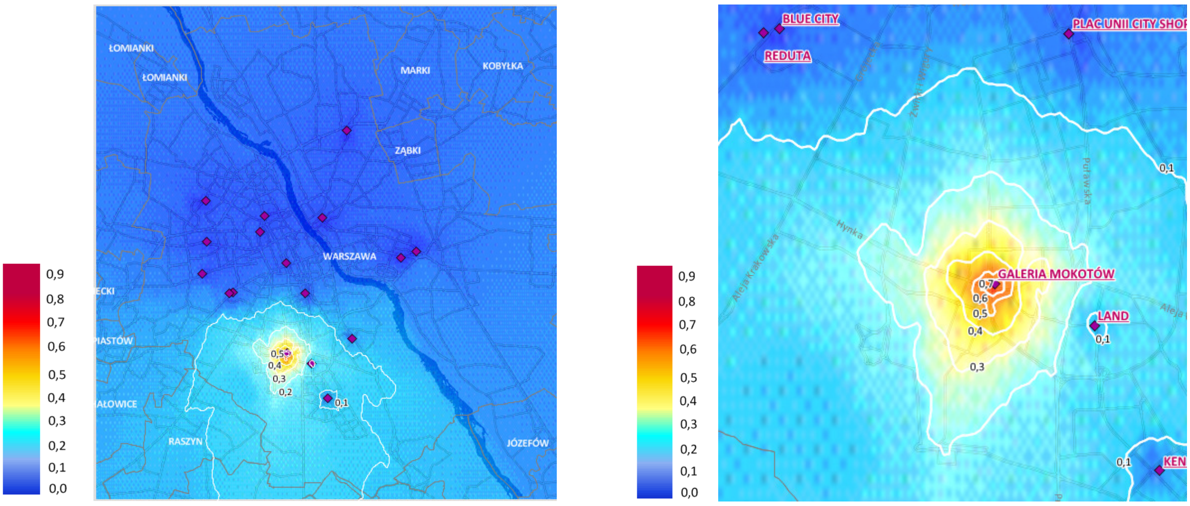

Fig. 3. Spatial coverage of Galeria Mokotów

Huff model relying on the distance measured along the road axis is not only more accurate but also leaves space in the analysis for incorporation of all natural obstacles which make it more difficult for clients to access a particular place. Galeria Mokotów is a case in point – the boundary of its market coverage coincides with the banks of the Vistula River, as a result of which its attractiveness for the residents to the right of the Vistula is next to none (see Fig. 3).

Then, based on the Lakshmanan and Hansen retail gravity model (formula 3, 4) the current market share was define and the changes that will occour after the appearance of new centers. The new centers affect all the shopping malls in the Warsaw area, but the influence is bigger for those located closer. In the map below you can see how big influence it has on the three centers located in the nearest neighbourhood (Fig. 4). The Table 2 shows the values for which will reduce the income of individual shopping centers (in relation to the situation before the appearance of new centers).

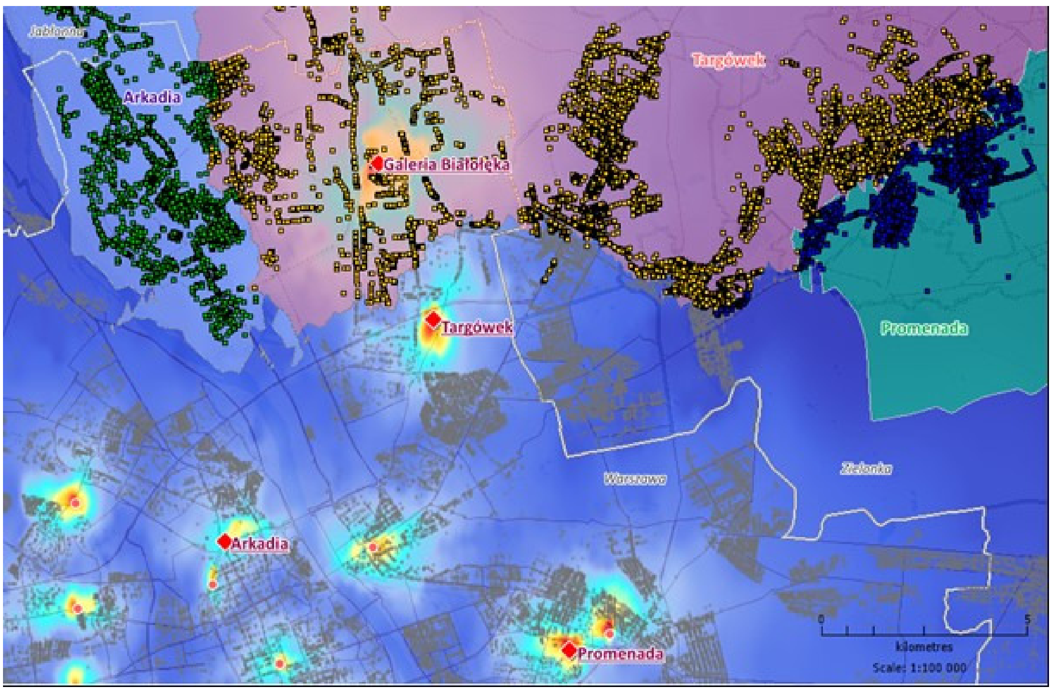

Fig. 4. Areas that before the advent of the Gallery Białołęka were in the ranges of the other centers

Based on the predictive Huff model found, that after the appearance of new trading center, the population which was chosing Arkadia, Targówek or Promenada so far, most likely decide to purchase in the Gallery Białołęka.

Conclusions

The emergence of new shopping centres in Warsaw noticeably affected the financial results of the other previously established centres. Each of the latter experienced a loss of 8–10%. The biggest changes were observed in the points of sale in direct neighbourhood of the planned ventures: forecast of a loss of over 10% for CH Targówek is due to its close vicinity to Galeria Białołęka, which is under construction nearby; while the loss of 10.24% predicted for KEN Center and that of 9.75% for Sadyba Best Mall are an outcome of Galeria Wilanów, which is being built at Przyczółkowa street.

Huff model and its variations are not only means to examine specific location and its potential for expansion or changes in the market occasioned by emergence of new competitors in a particular trade area. In fact, the algorithm suggested by Huff is frequently an important element in estimations of financial results of the existing outlets, it is part of sales forecasts or production and distribution plans in the analysed network.

The universality of the model and the ease with which it can be implemented are the reasons why it is gaining in popularity. The information obtained upon the analysis is supposed to support the decision-making process and contribute to taking economically-viable decisions. Yet, it should be kept in mind that both these and other results presented in this study should be treated as one of many parts of a decision-making process; they should never be taken unambiguously, as an absolute indicator of the market situation at the time being.

It seems natural that GIS applications are used in the process of physical environment planning. Not only do they have complex mechanisms for data feeding, editing, checking and updating or functions for level-specific integration of data of different formats and from different sources, but they also – and most importantly – come with instruments enabling profound spatial data analyses.

Huff model, as presented in this study, is just one of those which facilitate not only visualisation of the existing phenomena, but also – with proper factors and databases taken into consideration – prediction of changes which will occur in the physical environment if specific conditions are satisfied. The universality of the model and the ease of its implementation explain its growing popularity. The information acquired as a result of the analysis performed is to support decision-making processes and contribute to reasonable planning. Yet, what should be remembered is that both these and the other results presented in this study should be treated as one of many elements of a decision-making process and never should they be interpreted as unambiguous, because they do not demonstrate the current situation in the market in absolute terms.

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