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Optimization of a Grid of Candidates in the Search Procedure of the MAFA Method

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Abstract. Precise, carrier phase-based positioning requires a search procedure for "fixed solution" i.e. a solution, that takes into account the integer nature of ambiguities. In the classical approach (the Lambda method) the search for a fixed solution is conducted in the ambiguity domain. The computational load in this case depends on a number of satellites (the dimension of an ambiguity space amounts to: the number of satellites minus one). Conversely, in the MAFA method the search procedure is conducted in the coordinate-domain, (i.e. in three-dimensional space). It considerably reduces the computational load. In the article the search procedure in the coordinate domain is described. The technique of forming the optimized grid of candidates is presented, and the results of the tests are presented and analyzed.

Keywords: mixed integer-real least squares, GNSS data processing, Voronoi cell.

Conference topic: Technologies of Geodesy and Cadastre.

Introduction

Along with the development of the GNSS systems more and more satellites appear. At the same time the need arises for obtaining precise positions in real time. Therefore, the research on optimization of the computational process of precise positioning is now of particular importance. Recently a lot of research has been focused on this problem, e.g. (Teunissen 1995; Teunissen *et al.* 1996; de Jonge, Tiberius 1996; Teunissen, Kleusberg 1998; Kim, Langley 2000; Leick 2004; Chang *et al.* 2005; Hofmann-Wellenhof *et al.* 2008; Jazaeri *et al.* 2012).
In the case of classical methods (e.g. Lambda), the increasing number of satellites results in greater computational load. This is especially noticeable at the stage of the search procedure of the ambiguity resolution process. Therefore, it is proposed to apply another approach to GNSS data processing. In this approach the search procedure is performed in 3-dimensional coordinate space instead of in multidimensional ambiguity space. Thus, the dimension of search space does not depend on the number of satellites. It considerably reduces the computational load. The subject of this paper is the Modified Ambiguity Function Approach (MAFA) – the method of carrier phase data processing for precise positioning. The idea of this method is based on the Ambiguity Function Method (AFM), first proposed by Remondi (1984, 1989). A detailed description of the MAFA method can be found in (Cellmer *et al*. 2010; Cellmer 2011, 2012, 2013, 2014; Cellmer *et al.* 2013; Kwasniak *et al.* 2016)). In this method ambiguities are not computed explicitly, although the results take into account their integer nature. The optimisation of the search procedure that is the part of the MAFA method in this paper is proposed. The need of a good approximate position is the main problem when using the MAFA method. The MAFA method can provide the correct solution only if the *a'priori* position is inside the space bounded by a certain surface (Voronoi cell). This necessary condition of obtaining a correct solution was analysed by Cellmer (2011). But there are occasions where the approximate coordinates are known with accuracy of a few centimeters, e.g. the surveying network for deformation monitoring (e.g. Nowel 2016a, 2016b). Unfortunatelly, usually obtaining such a good approximate coordinates is not possible. However, an appropriate search procedure can provide the good approximate position. The optimization of such a procedure is the main problem of this paper. In the next section the MAFA method is described. Then, the search procedure in the coordinate domain is described. Next, the discussion about the density of a grid of candidates is presented. A new way of forming such a grid is proposed. The results of the tests are discussed and analysed. Finally, some conclusions are formulated.

The MAFA method

The model of double differenced (DD) carrier phase observations is written as:

$$\mathbf{e} = -\frac{1}{\lambda}\mathbf{B}\mathbf{b} + \boldsymbol{\delta} \tag{1}$$

with:

$$\boldsymbol{\delta} = \left(\boldsymbol{\Phi} - \tfrac{1}{\lambda}\boldsymbol{\rho}(\mathbf{x}^0)\right) - round\left(\boldsymbol{\Phi} - \tfrac{1}{\lambda}\boldsymbol{\rho}(\mathbf{x}^0)\right), \tag{2}$$

where:

- $\mathbf{e}$ – error vector *(n×1)*,
- $\mathbf{b}$ – parameter vector (increments to *a priori* coordinates vector $\mathbf{x}^0$),
- $\mathbf{B}$ – design matrix *(n×3)*,
- $\boldsymbol{\delta}$ – vector of misclosures *(n×1)*,
- $\boldsymbol{\Phi}$ – vector of DD carrier phase observations (in cycles)
- λ – signal wavelength
- *round*(.) – function of rounding to the nearest integer
- $\mathbf{x}^0$ – vector of approximate coordinates of the receiver
- $\boldsymbol{\rho}(\mathbf{x}^0)$ – vector of DD approximate geometrical ranges (DD geometric distance computed using *a priori* position and satellite coordinates).

For simplicity, it is assumed in this paper, that there are only three real parameters in the model: the coordinates of the receiver. However, it is possible to add more real parameters. The model (1) together with the least squares (LS) objective function:

$$\boldsymbol{\Psi} = \mathbf{e}^T\mathbf{P}\mathbf{e}, \tag{3}$$

with $\mathbf{P}$ standing for the weight matrix, define the adjustment problem.

The estimates of $\mathbf{b}$ and $\mathbf{e}$ can be obtained as follows:

$$\breve{\mathbf{b}} = \lambda\left(\mathbf{B}^T\mathbf{P}\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{P}\boldsymbol{\delta}\,; \tag{4}$$

$$\breve{\mathbf{e}} = -\tfrac{1}{\lambda}\mathbf{B}\breve{\mathbf{b}} + \boldsymbol{\delta}\,. \tag{5}$$

It was proven by Cellmer *et al*. (2010) that although there are no ambiguity parameters in model (1), their integer nature is taken into account in the final solution.

The search procedure

In general case, we can not assume that the approximate position is good enough to satisfy the necessary condition. Therefore, it is proposed to carry out the search procedure. In the MAFA method the search procedure can be performed in the coordinate domain instead of in the ambiguity domain, like in classic method (e.g. Lambda). The search region in the coordinate domain can be formed as the ellipsoid error of the approximate position (Fig. 1a).

a) Parameters of error ellipsoid b) Set of candidates

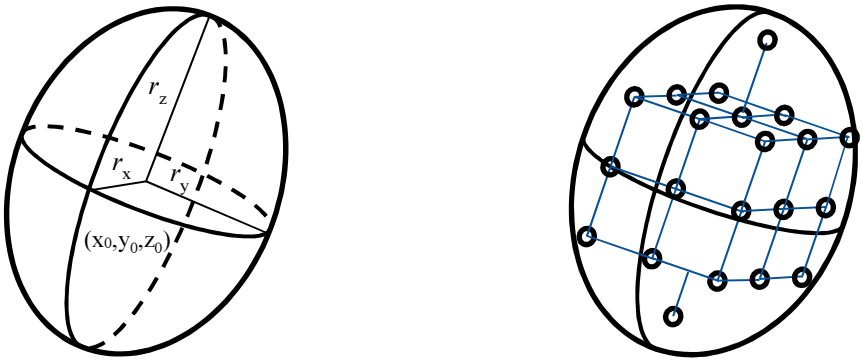

Fig. 1. Search region and grid of candidates

This approximate position can be obtained from any technique, which provides the coordinates of the position together with their covariance matrix. Inside the ellipsoid error, a grid of candidates is formed (Fig. 1b).

The following parameters determine the size and the shape of the ellipsoid error: the centre of the ellipsoid, the length of its major axes, the orientation of the axes. These values are computed based on the coordinate vector of the

approximate position (x^0) and their covariance matrix (Q_x). The centre of an ellipsoid is at the point of the approximate position. The axes' lengths are computed using the formula:

$$r_x = \hat{\sigma}_0 \sqrt{\frac{3F_c}{\mu_x}},\ r_y = \hat{\sigma}_0 \sqrt{\frac{3F_c}{\mu_y}},\ r_z = \hat{\sigma}_0 \sqrt{\frac{3F_c}{\mu_z}}, \tag{6}$$

where:

$\hat{\sigma}_0$ – variance factor for the approximate position

F_c – critical value of the F-distribution variable with 3 and n-3 degrees of freedom and assumed confidence level

μ_x, μ_y, μ_z – eigenvalues of $\mathbf{Q}_x^{-1}$ matrix.

The orientation of the ellipsoid error is governed by the orthonormal **S** matrix, which is used in the eigenvalue decomposition of the symmetric matrix $\mathbf{Q}_x^{-1}$:

$$\mathbf{Q}_x^{-1} = \mathbf{S}^T \mathbf{M} \mathbf{S}, \tag{7}$$

where: **M** is the diagonal matrix of eigenvalues.

The grid of candidates is formed inside the ellipsoid error, in the local coordinate system, which is oriented in line with the orientation of the main axes of an ellipsoid error. Then the points of the grid are transformed into the geocentric GNSS coordinate system. The dependency between coordinate vector $\mathbf{x_s}$ in the local coordinate system associated with ellipsoid error and the coordinate vector **x** in the geocentric coordinate system is:

$$\mathbf{x}_s = \mathbf{S}(\mathbf{x} - \mathbf{x}^0). \tag{8}$$

For each candidate, the value:

$$\breve{\Psi} = \breve{\mathbf{e}}^T \mathbf{P} \breve{\mathbf{e}} \tag{9}$$

is tested. It is assumed that the final solution minimizes this value. A matter to be determined is the density of the grid of candidates. It is assumed that the density of the grid points depends on the wavelength of the carrier phase data used as an observation set. The density of the grid of candidates should be established in such a way as to not omit the Voronoi cell (VC) of the correct solution. The VC of point **z** in a lattice is the set of points in space that are closer to **z** than any other point in the lattice (Hassibi, Boyd 1998). The term "Voronoi cell" was also used in the topic of ambiguity resolution by Xu (2006). We used this term in the meaning of the region containing candidates which give the same solution. In the next section, a way of determining the optimal density of the search region is described.

In order to reduce the number of candidates, cascade processing can be employed (Han, Rizos 1996; Jung, Enge 2000; Urquhart 2009). In this approach, the solution is computed in several steps using observation sets with different wavelengths from the longest to the shortest. In the MAFA method two observation sets in cascade processing are applied: the widelane combination (in the first step): $L_{1,-1}$ = L1-L2 with the wavelength $\lambda_{1,-1}$ = 0.86m and L1-signals with the wavelength λ_1=0.19m in the second step of cascade processing.

Fixing the optimal density of the grid of candidates

It is obvious that the density of the grid of candidates depends on the size of the VCs, as well as on their arrangement inside the search region. It can be assumed that all the VCs are the same size and shape and they are distributed evenly in the search region (Baselga 2014). Moreover, it is assumed that the VCs are convex, and that they fill the whole space. The size of the VC depends on the wavelength of the carrier phase observation (or linear combination) used as data in the computational process. The shape of the VCs and their arrangement in space is unique for the satellite configuration. Therefore, it is impossible to determine the universal, optimal density for a grid of candidates which would be valid for each case. In this contribution, we propose a method of setting the density of a grid of candidates for the most unfavourable scenario of the VC shape and arrangement. The starting point for considerations is the distance between the centres of the two neighbour VCs. The problem is to determine the relationship between this distance and the density of the grid of candidates. Figure 2 depicts this relationship in the two-dimensional case. However, the presented example can be generalised into the three-dimensional case. The grid of squares depicts the VCs. It is assumed that the correct solution is at point A. Thus, to obtain the correct solution, at least one candidate of the grid should fall into the VC with the centre at point A. The distance between the centres of the two neighbour VCs is denoted as: a = AB.

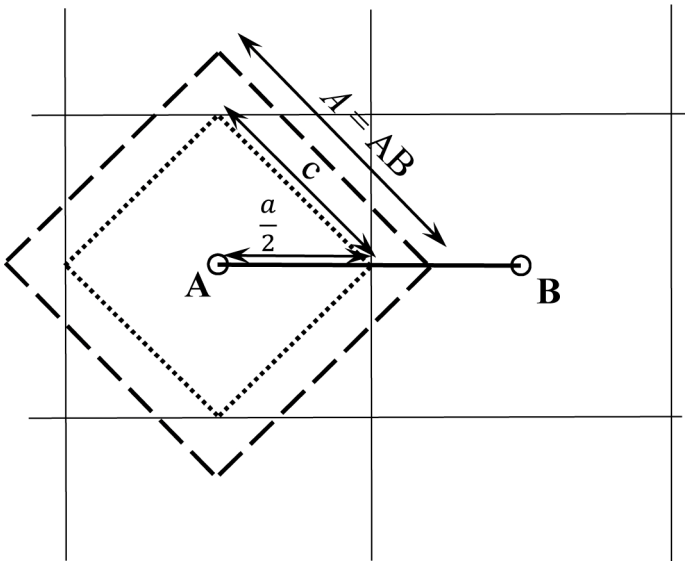

Fig. 2. Determining the minimum density of the grid

Let's assume that the density of a grid of candidates (expressed by the distance between two neighbour candidates) amounts to a (dashed line). In this case the VC with the centre at point A could be omitted. This is the case when the orientation of the grid of candidates does not correspond with the orientation of the axes of the VC (it should be noted that any orientation of the grid of candidates is possible – it only depends on the orientation of the ellipsoid error). This is depicted by the dashed line depicting the part of a grid of candidates. The vertices of this square depict the neighbouring candidates of the grid. None of them is inside the VC with the centre at point A. In general, the minimal density of the grid that avoid the risk of omitting the VC corresponds to the largest, arbitrarily oriented square, contained in the VC. This square is depicted by a dotted line. The length of the edge of this square is:

$$c = \frac{\sqrt{2}}{2} a. \tag{10}$$

This value is assumed as the density of a grid of candidates.

Experiment

The experiment was performed on a set of simulated observations on an actual, short baseline (about 70 m). Figure 3 depicts the location of this baseline.

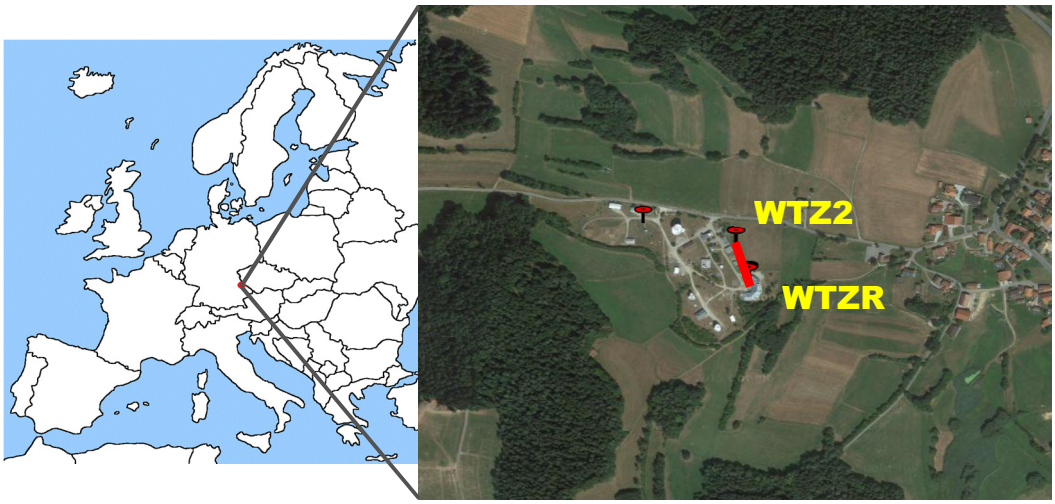

Fig. 3. The baseline location

Data were prepared for three satellite configurations. These satellite configurations are depicted in Fig. 4.

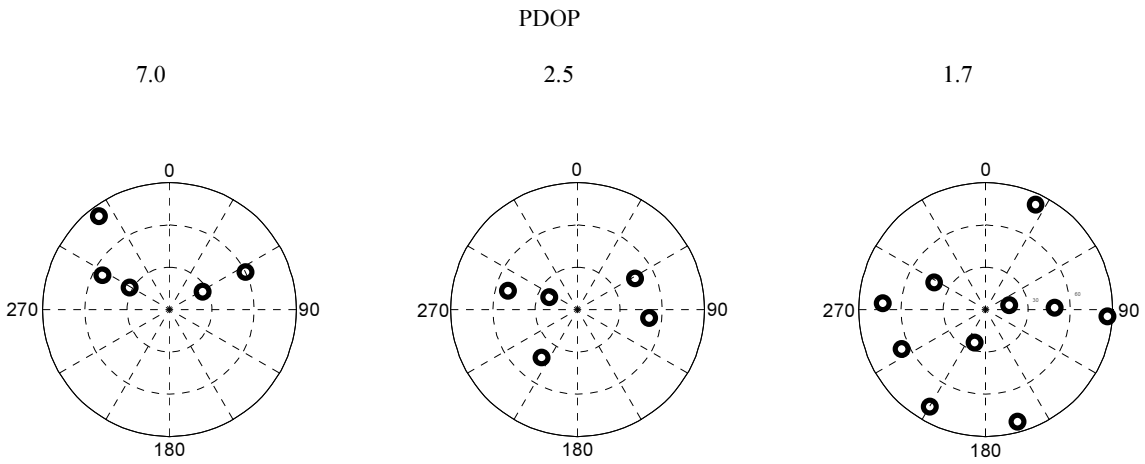

Fig. 4. The satellite configurations

The code and carrier phase observations were generated using the actual positions of the satellites and the receivers in selected short (3-epoch and 5-epoch) sessions. First the geometric distances were computed (from coordinates). Then, these distances were burdened by randomly generated errors. It was assumed that the errors were normally distributed with standard deviation $\sigma_c = 0.3$m for code observations, and $\sigma_p = 0.01$ cycle for carrier phase observations. The weight matrix of the DD carrier phase observations was formed without taking into account elevation dependent factors. It is an inverse of the following covariance matrix:

$$\mathbf{Q} = \sigma_p^2 \begin{bmatrix} 4 & 2 & \cdots & 2 \\ 2 & 4 & \cdots & 2 \\ \vdots & \vdots & \ddots & 2 \\ 2 & 2 & 2 & 4 \end{bmatrix}. \tag{11}$$

The purpose of the tests was setting the optimal density of a grid of candidates. At the beginning the grid was formed inside a cube 0.5m x 0.5m with a density (the distance between neighbour points of a grid) equal to 2 cm. The solutions obtained for each of the candidates were grouped according to belongingness to separated VCs. Each point from one group has the same solution – the centre of the VC. Thus, the new grid of points is formed. This grid is composed of the centres of all the VCs. Then, the distances between the point of the correct solution and each point of the new grid are computed. The minimal distance is the basis for determining the density of the grid of candidates. This process was repeated 1000 times for each case of satellite configuration, session length and linear combination of carrier phase observations. For the scenario with widelane data, the initial grid was formed inside a cube 1.5m x 1.5m with a density equal to 5 cm. Table 1 contain the mean values and standard deviations of the minimal distance computed from 1000 repetitions for each case.

Table 1. Minimal distances between centres of neighbouring Voronoi cells (mean values computed from 1000 repetitions)

Data	PDOP					
	7.0		2.5		1.7	
	3-epoch	5-epoch	3-epoch	5-epoch	3-epoch	5-epoch
L_1	0.20 (±0.03)	0.22 (±0.04)	0.14 (±0.03)	0.13 (±0.01)	0.08 (±0.01)	0.08 (±0.01)
L_1-L_2	1.04 (±0.24)	0.93 (±0.19)	0.57 (±0.04)	0.52 (±0.03)	0.32 (±0.01)	0.31 (±0.01)

It is proposed here to assume the final value of the minimal distance between two centres of neighbour VCs as lower bound of the 0.95 confidence region of the normally distributed variable:

$$d_{min} = d_{sr} - 2\sigma = a,$$

where: d_{sr} is the mean value and σ is standard deviation. It is interpreted as the a value from the formula (10) and Fig. 2.

Table 2 contains d_{min} values and Table 3 the distances between neighbour points in the grid.

Table 2. Minimal distances between centres of neighbouring Voronoi cells (mean value minus doubled value of standard deviation)

Data	PDOP					
	7.0		2.5		1.7	
	3-epoch	5-epoch	3-epoch	5-epoch	3-epoch	5-epoch
L_1	0.14	0.14	0.08	0.11	0.06	0.06
L_1-L_2	0.56	0.55	0.49	0.46	0.30	0.29

Table 3. Density of the grid of candidates (distances between neighbour points in the grid)

Data	PDOP					
	7.0		2.5		1.7	
	3-epoch	5-epoch	3-epoch	5-epoch	3-epoch	5-epoch
L1	0.10	0.10	0.06	0.08	0.04	0.04
L1-L2	0.40	0.39	0.35	0.33	0.21	0.21

The values in Table 3 are obtained by multiplying respective values from Table 2 by $\frac{\sqrt{2}}{2}$ (the c value in Fig. 2).

As can be seen the distance between neighbour points in the grid for widelane data ranges from 0.21 (PDOP = 1.7, 5-epoch session) to 0.40 m (PDOP = 7.0, 3-epoch session). For L1-only data these values amount to 0.04m (PDOP = 1.7, 5-epoch session) and 0.10 m (PDOP = 7.0, 3-epoch session).

Conclusions

The maximum distance between points in the grid of candidates which at the same time allows that at least one candidate falls into each VC is assumed as the optimal density of the grid of candidates. It is obvious that the density of the grid of candidates for widelane data is much lower than for L1 data. Moreover, as can be seen, the optimal distance between points in the grid strongly depends on satellite configuration. It is worth noting that the smallest distance between points in the grid should be fixed in the case of the best satellite configuration. It can be explained by more constraints determining point position in the case of more satellites with a good geometrical configuration. In this case, the VC is bounded by a greater number of walls, and therefore shrinks its volume.

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