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A Minimum Size of the Search Cube in the MAFA-ILS Method

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Abstract. The Modified Ambiguity Function Approach – Integer Least Squares (MAFA-ILS) is one of the modern method for the precise real-time GNSS positioning, for many applications such as geodesy or surveying engineering. Contrary to the classic approach, in the MAFA-ILS method the solution is sought in the coordinate domain. However, to obtain a proper solution, the search region cannot be too small. On the other hand, to have an effective solution – in the computational load sense – this region cannot be too big. In this case the determination of the minimum size of appropriate search region is not a trivial task. The paper presents a certain solution of this problem.

Keywords: GNSS positioning, ambiguity resolution, LAMBDA, Voronoi cell.

Conference topic: Technologies of Geodesy and Cadastre.

Introduction

Precise GNSS positioning is used for many applications such as geodesy, surveying engineering or precise navigation. It is known that positioning requires resolving the so-called mixed integer-real problem. That is, to determine a precise position it is additionally necessary to consider/determine integer ambiguities of the carrier phase GNSS observations. Hence not only real-valued coordinates are unknowns in this case, but also the ambiguity integer is unknown. The ambiguity integer estimation is only the basic problem.

It is well known that estimation of the integer ambiguities is a non-standard problem, because the classical theory of estimation is invalid here. This problem has been a rich source of GNSS research over the last thirty years. Teunissen developed and ordered the entire family of ambiguity estimators, and not only the integer ones (e.g. Teunissen 1999a). The solution search proceeds here in the *ambiguity domain*. The second category of algorithms includes the very first ambiguity estimation technique developed, namely the Ambiguity Function Method (AFM). This method was first introduced by Counselman and Gourevitch (1981). The solution search proceeds here in the *coordinate domain*. The last clear category includes the simplest techniques that use C/A-code or P code pseudoranges directly to estimate the ambiguities of the corresponding carrier phase observations (e.g., Hofmann-Wellenhof *et al.* 2008). Computations proceed in the *measurement domain*.

From a probabilistic point of view, the so-called Mixed Integer-Real Least Squares (MIRLS) estimation is the best solution here (Teunissen 1999b). This solution correspond to the first category of algorithms, hence computations proceed in the *ambiguity domain*. The computational efficiency of this solution is satisfactory for real-time GNSS positioning. Hence the MIRLS estimation is today widely used for geodesy or surveying engineering. However, this applies to only a few or a dozen or so ambiguity cases. In the case of a larger number of ambiguities (two or more satellite systems, e.g., Paziewski and Wielgosz (2014), the computation time may be too long for real-time applications. It is still a challenge for GNSS investigators.

Therefore, the paper Nowel *et al.* (2016) presents an alternative approach to the MIRLS estimation (MAFA-ILS method). This approach is a variant of the Modified Ambiguity Function Approach (MAFA), from the AFM class (Cellmer *et al.* 2010; Cellmer 2011). Integer ambiguities are not explicitly estimated here, however their integer nature is preserved and may be optionally calculated. The solution is sought in three-dimensional *coordinate domain*, instead of *n*-dimensional *ambiguity domain*. Because the dimension of search space is constant here, the computational efficiency does not depend as much on the number of satellites, as in conventional MIRLS estimation. The numerical tests have shown that the reliability and precision of results from both approaches to MIRLS estimation are equivalent.

However, the MAFA-ILS method still has open non-trivial scientific problems. The determination of the shape and of the minimum size of appropriate search space is one of them. An appropriate search space is such a space, in which the sought solution will certainly exist. From the computational efficiency point of view this space should be as small as possible. This paper is exactly related to the problem of defining the minimum size of the appropriate search space in the MAFA-ILS method.

Mixed Integer-Real Least Squares Estimation

This section presents the theoretical basis for the MIRLS-estimation. Here discussion is limited to only those issues, which shall be significant for the further part of the paper.

The double difference (DD) carrier phase observation equation may be represented as:

$$\mathbf{\Phi} = \boldsymbol{\rho} / \lambda + \mathbf{a} + \mathbf{e}\,, \tag{1}$$

where $\boldsymbol{\rho}$ is the vector of DD geometrical distances, λ is the wavelength of signal, **a** is the vector of DD carrier phase ambiguity and **e** is the vector of DD carrier phase measurement noise. Linearization of the observation equations, with respect to the unknown parameters, gives the well-known mixed integer-real linear model:

$$\mathbf{y} = \mathbf{A}\mathbf{a} + \mathbf{B}\mathbf{b} + \mathbf{e},\ \mathbf{a} \in Z^n, \mathbf{b} \in R^3\,, \tag{2}$$

where **y** is the n-dimensional (mn-dimensional for m epochs) vector of "observed minus computed" DD carrier phase observables, **A** and **B** are the appropriate design matrices, **b** is the vector that contains the increments of the unknown baseline components, Z^n is the n-dimensional space of integers and R^3 is the three-dimensional space of reals.

In order to solve for this system of equations, the LS principle is applied:

$$\breve{\mathbf{e}}^{\mathrm{T}} \mathbf{Q}_y^{-1} \breve{\mathbf{e}} = \min\,, \tag{3}$$

where $\breve{\mathbf{e}}$ (residuals) is the LS estimate of **e**, but conditioned on $\mathbf{a} \in Z$. Since the ambiguities are known to be integer, one is dealing with the MIRLS problem rather than a standard LS problem. The quadratic objective function of the above problem can be decomposed into a sum of two squares (e.g., Teunissen 1995):

$$\hat{\mathbf{e}}^{\mathrm{T}} \mathbf{Q}_y^{-1} \hat{\mathbf{e}} + \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right)^{\mathrm{T}} \mathbf{Q}_{\hat{a}}^{-1} \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right) = \min\,. \tag{4}$$

The proof can be found in Cai *et al.* (2009). The MIRLS problem (3) may be solved in three steps. The first step consists of solving ordinary unconstrained LS problem, i.e.:

$$\text{Step 1 (float solution): } \hat{\mathbf{e}}^{\mathrm{T}} \mathbf{Q}_y^{-1} \hat{\mathbf{e}} = \min\,. \tag{5}$$

As a result of this step, real-valued estimates for both the ambiguities, $\hat{\mathbf{a}}$, and the baseline components, $\hat{\mathbf{b}}$, are obtained, together with their corresponding cofactor matrices. This result forms the input for the second step. In the second step one solves the ILS-estimator of the ambiguity vector, $\breve{\mathbf{a}}$. It follows from solving:

$$\text{Step 2 (ILS solution): } \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right)^{\mathrm{T}} \mathbf{Q}_{\hat{a}}^{-1} \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right) = \min\,. \tag{6}$$

Once the ILS-estimator of ambiguity vector is available, $\breve{\mathbf{a}}$, the final constrained baseline solution is obtained as:

$$\text{Step 3 (fixed solution): } \breve{\mathbf{b}} = \hat{\mathbf{b}}\left(\breve{\mathbf{a}}\right) = \hat{\mathbf{b}} - \mathbf{Q}_{\hat{b}\hat{a}} \mathbf{Q}_{\hat{a}}^{-1} \left(\hat{\mathbf{a}} - \breve{\mathbf{a}}\right) \tag{7}$$

together with their corresponding cofactor matrix.

The ILS-estimation (6) is the basic task in the MIRLS-estimation (2), (3). Because of the presence of the integer constraint $\mathbf{a} \in Z^n$ there are unfortunately no standard techniques available for solving ILS problem as they are available for solving ordinary LS problems. The real-valued LS-estimator (5) is usually highly correlated and its confidence region (hyper-ellipsoid) is usually extremely elongated, hence finding a problem (6) solution may be extremely time-consuming. Therefore, the ambiguity search space is most often reduced to increase the computational efficiency of ILS-estimation. The integer 'decorrelation' of real-valued LS-estimator of ambiguity vector and its cofactor matrix plays a significant role here. The new virtual ambiguities are almost independent and the search for the solution of (6) is then relatively quick. Because of the present study limits, the search space reduction algorithms (e.g., LAMBDA algorithm) and search algorithms shall not be discussed here. This may be found *inter alia* in previously quoted papers.

Modified Ambiguity Function Approach – Integer Least Squares

The MAFA method assumes that

$$\mathbf{a} = round(\mathbf{\Phi} - \boldsymbol{\rho} / \lambda), \tag{8}$$

where *round* is a function of rounding to the nearest integer value. After vector (8) substitution to Eq. (1) one obtains the following modified DD equation of carrier phase observations:

$$\mathbf{\Phi} = \boldsymbol{\rho} / \lambda + round(\mathbf{\Phi} - \boldsymbol{\rho} / \lambda) + \mathbf{e}. \tag{9}$$

It is easy noticeable that only the receiver coordinates are the unknown parameters in this equation. After this equation linearization, one obtains a linear functional model of the MAFA method:

$$\boldsymbol{\delta} = \mathbf{Bb} + \mathbf{e},\ \mathbf{b} \in R^3, \tag{10}$$

where $\boldsymbol{\delta} = \left(\mathbf{\Phi} - \boldsymbol{\rho}^0 / \lambda\right) - round\left(\mathbf{\Phi} - \boldsymbol{\rho}^0 / \lambda\right)$, $\boldsymbol{\rho}^0 = \rho\left(\mathbf{x}^0\right)$ is the vector of computed DD geometrical distances and $\mathbf{x}^0$ is the vector of receiver approximate coordinates. After adding the optimisation condition of LS method (3) to Eq. (10) one obtains the optimisation problem of the MAFA method. The solution of that problem is the following estimator:

$$\breve{\mathbf{b}} = \lambda\left(\mathbf{B}^{\mathrm{T}}\mathbf{Q}_y^{-1}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}_y^{-1}\boldsymbol{\delta} \tag{11}$$

together with their corresponding cofactor matrix. After linearization, Eq. (8) now has the form:

$$\mathbf{a} = round\left(\mathbf{\Phi} - \boldsymbol{\rho}^0 / \lambda\right). \tag{12}$$

It is easily noticeable, that the MAFA method can provide correct solutions only when the receiver approximate coordinates, $\mathbf{x}^0$, fulfill the condition (12). So-called ‘good’ Voronoi cell (Cellmer 2012) is a graphical interpretation of this condition in the *coordinate domain*. Approximate coordinates, which are situated in this cell, i.e. those meeting condition (12), are ‘good’ approximate coordinates. The entire *coordinate domain* is filled **-** without gaps or overlaps – with Voronoi cells. Figure 1 presents an example of a few Voronoi cells in 2D space.

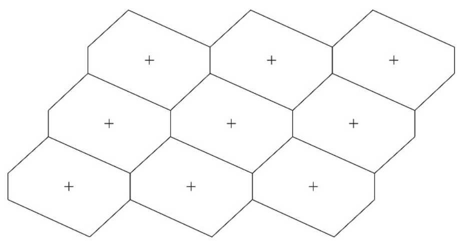

Fig. 1. Example of some Voronoi cells in the 2D space

These cells were empirically generated based on Eq. (12) from a dense set of coordinates, for some functional model (10). Condition (12) is a necessary condition and at the same time a sufficient condition to obtain a correct solution in the MAFA method. The determination of ‘good’ coordinates, i.e. such that meet this condition, is the basic problem here (e.g., Cellmer 2013, 2015; Kwaśniak *et al.* 2016). But there are occasions where the approximate coordinates are ‘good’, e.g. a surveying network for deformation monitoring (e.g., Duchnowski, Wiśniewski 2014; Nowel, Kamiński 2014; Nowel 2015; Wiśniewski, Zienkiewicz 2016). However, it is not possible to always determine ‘good’ approximate coordinates. There is mostly small risk that these coordinates may be ‘bad’.

However, another solution is possible here. Namely, it is always possible to find such initial approximate coordinates, which shall give the ILS solution, i.e. which shall be the optimisation problem (10), (3) solution. Such solution has been conventionally named MAFA-ILS method (Nowel *et al.* 2016). In the context of MIRLS estimation (5), (6), (7) and without loss of generality, the course of calculations in the MAFA-ILS solution may be generally presented as follows (Fig. 2):

Step 1: Estimation $\hat{\mathbf{b}}$ ⟹ Step 2: Search $\breve{\mathbf{b}}$

e.g. (2), $\hat{\mathbf{e}}^{\mathrm{T}}\mathbf{Q}_y^{-1}\hat{\mathbf{e}} = \min$ (10), $\breve{\mathbf{e}}^{\mathrm{T}}\mathbf{Q}_y^{-1}\breve{\mathbf{e}} = \min$

Fig. 2. Diagram of the algorithm of the MAFA-ILS solution

The first step is intended only to provide initial approximate coordinates, to determine the search region centre. In the paper Nowel *et al.* (2016) float coordinates are suggested, as is the case in the MIRLS estimation. However, step 2 has the basic importance. This step is intended at finding such approximate coordinates, which shall give the optimisation problem (10), (3) solution. These coordinates are called 'ILS' approximate coordinates.

A trivial search procedure may consist of generating an appropriate cloud of approximate coordinates (candidates) and showing such coordinates, which are 'ILS' approximate coordinates. Such cloud of point must obviously contain at least one set of 'ILS' approximate coordinates. The paper Nowel *et al.* (2016) suggested an appropriate cube of grid points around float coordinates, $\hat{\mathbf{x}}$. Such a set of points, may be easily generated e.g. by means of the recurrent formula which presented in the above paper.

Now it is necessary to find 'ILS' approximate coordinates in this set of candidates. A most trivial way may consist in calculating a square form (3) for each candidate, $\mathbf{x}_i^0$, with respect to the functional model (10). The candidate, which will give the smallest value of this form, will be the 'ILS' approximate coordinates. For these coordinates, ambiguity values (12) will be the same for all *m* epochs and equal to ILS ambiguity estimator (6).

The minimum size of the search cube

Now, the key question is: What could be the minimum size of search cube and the minimum grid points density in this region so that the candidates set would contain at least one set of 'ILS' approximate coordinates? It may be noticed that to satisfy the above condition at least one candidate must 'fall into' each Voronoi cell. So the grid points' density (the distance between neighbouring candidates, *d*) depends on the Voronoi cells' size and shape. The paper Nowel *et al.* (2016) suggested a certain solution for the value of the minimum grid points density. Besides, our empirical tests show that if $d = 0.5\lambda$, then at least one candidate is present in each Voronoi cell. Instead, the minimum size of appropriate search cube is much more difficult task.

It is noticeable, that the minimum size of appropriate search cube side, *s*, depends on the distance between the 'ILS' Voronoi cell and the search region centre (in this case float coordinates). Hence, this problem may be resolved empirically based on Monte Carlo (MC) simulations. Specifically, around some unique 'ILS' solution (centre of the unique 'ILS' Voronoi cell) it is possible to generate – for a specific model (10) – a cloud of float coordinates, which correspond to this unique solution. Obviously, for each unique solution the cloud shall have the same shape and size. Based on such a cloud it is possible to determine coordinate differences between this solution and the float points, which correspond to this unique solution. On the basis of such differences it is now possible to determine e.g. the minimum length of appropriate search cube side. It is easy noticeable that this cube side should not be smaller than a double value of the largest coordinate difference between the unique 'ILS' solution and the float point, which correspond to this unique solution, i.e.

$$s \geq 2 \cdot max\left(max\left(abs\left(\hat{\mathbf{X}} - \breve{\mathbf{X}}\right)\right)\right), \tag{13}$$

where $\hat{\mathbf{X}}$ is the float coordinates' matrix, which correspond to the unique 'ILS' solution and $\breve{\mathbf{X}}$ is the coordinates' matrix of the unique 'ILS' solution (the same in each column). Then, the 'ILS' approximate coordinates (any part of the 'ILS' Voronoi cell) shall be in the search cube with probability:

$$P = 1 - 1/N, \tag{14}$$

where *N* is the number of simulations. Figure 3 shows some unique 'ILS' Voronoi cell (in the centre), the corresponding float points cloud (N = 5000) and the search cube appropriate here.

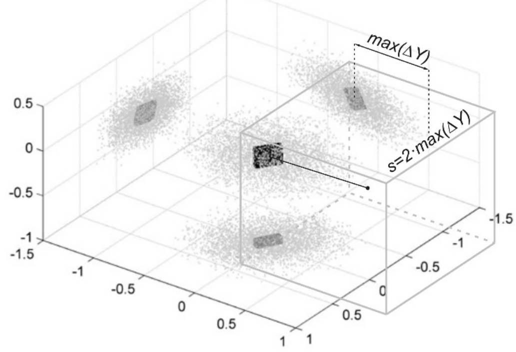

Fig. 3. Example of some "ILS" Voronoi cell, their "ILS" float points cloud and the appropriate search cube [m]

In this case the maximum coordinate difference, *max*(ΔY), was 0.68 m, hence the minimum length of appropriate search cube side was $s \geq 1.36\text{ m}$ (13).

The above empirical way of determining the minimum size of appropriate search cube concerns a specific constellation of GNSS satellites. This is obvious. However, this approach may be applied to formulate a certain universal solution. Namely, for many various configurations of satellites the values of (13) can be calculated and then the relationships between those values and the values characterizing individual satellite configurations, e.g. PDOP values, may be determined. In this way a universal formula for calculation of the minimum size of appropriate search cube for any satellite configuration can be empirically determined. The preliminary tests are presented in the next section.

Numerical example

The numerical test is based on two simulated three epoch observational sessions (interval of 120 s) from six and eleven actual satellites for short actual baseline ($\sim 70\text{ m}$) located in south Germany (Fig. 4).

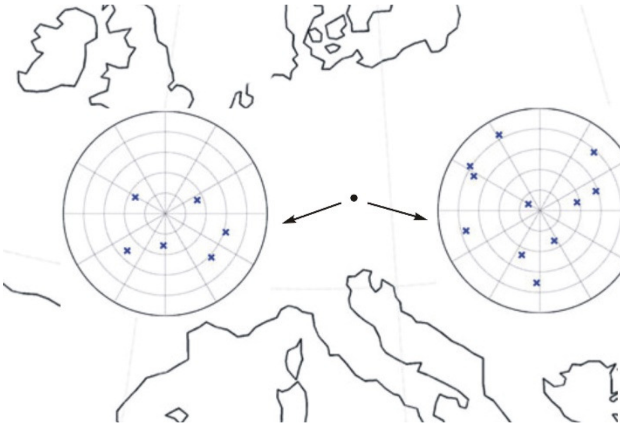

Fig. 4. Skyplot of GPS constellations as used in the simulations, two scenarios; average: PDOP = 2.5 (left) and good: PDOP = 1.7 (right)

GPS single frequency L1 signals ($\lambda = 0.1903\text{ m}$) are used. A theoretical coordinates were used for computation of the theoretical DD geometric distances. The noise of the simulated DD carrier phase observations is assumed as zero mean Gaussian noise, $\mathbf{e}(mn \times 1) \sim N\left(\mathbf{0}, \sigma^2 \cdot diag\left(\ldots \;\; \mathbf{Q}_{y,j} \;\; \ldots\right)\right)$, where

$$\mathbf{Q}_{y,j}(n \times n) = \begin{bmatrix} 1 & \cdots & 0.5 \\ \vdots & \ddots & \vdots \\ 0.5 & \cdots & 1 \end{bmatrix}$$

is the cofactor matrix for the subset of DD carrier phase observations from the *j*th epoch and σ^2 is the variance of the simulated DD carrier phase observations. The MC method has been used for this purpose. Independent random variables vectors, $\mathbf{e}'$, were thus obtained. Then the vectors were changed into dependent random variables' vectors by means of the following linear transformation

$$\mathbf{e} = \left(chol\left(\mathbf{Q}_y\right)\right)^{\mathrm{T}} \cdot \mathbf{e}',$$

where $chol(\bullet)$ is a upper triangular matrix from the Cholesky decomposition. The above error vectors were generated for the DD observations' noise $\sigma = 0.02$ cycle. The theoretical DD geometric distances have been burdened by simulated errors, **e**, and some integer values of the ambiguities, **a**. The data processing was performed with the application of the MAFA-ILS method (10), (3). The float coordinates were the initial approximate coordinates (search region centre). In addition, for cognitive purposes, the data processing was performed with the application of the MIRLS estimation (2), (3). The MIRLS estimation was implemented by means of the LAMBDA method.

The test was intended mainly for comparing values of the minimum size of appropriate search cube (13) for average and good constellation (Fig. 4). The values of the minimum grid points density were calculated based on formula from paper Nowel *et al.* 2016. These calculations gave $d = 0.071$ m and $d = 0.057$ m, for average and good constellation, respectively. However, these are very conservative values. Whereas, the minimum size of the appropriate search cube were calculated empirically based on formula (13). These calculations gave $s = 1.684$ m and $s = 1.172$ m, for average and good constellation, respectively. For both scenarios 5,000 simulations were performed. Figure 5 presents the clouds of the 'ILS' float coordinates.

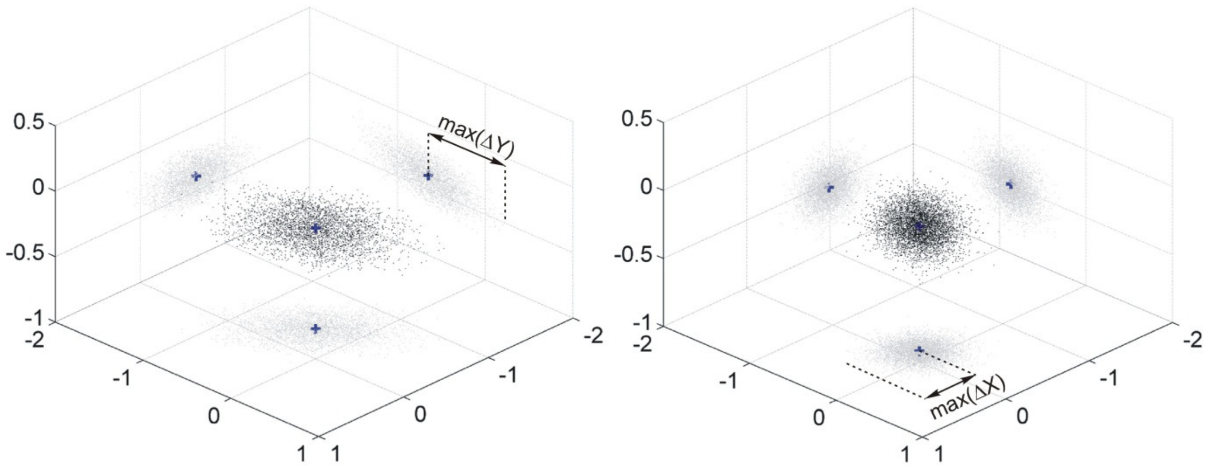

Fig. 5. The clouds of the 'ILS' float coordinates [m]

The maximum coordinate difference was $max(\Delta Y) = 0.842$ m and $max(\Delta X) = 0.586$ m, for the average and good constellation, respectively. In general, the above results show that a stronger functional model (10) produces a smaller value of maximum coordinate difference in the cloud of the 'ILS' float coordinates and, consequently, the smaller minimum size of appropriate search cube (13) may be taken. Hence a conclusion can be drawn that there is a relationship between the minimum search cube size and the satellites' constellation geometry.

In addition, Table 1 presents the relationship between the success rate values of the ambiguity estimation, *SR*, and the different values of the search cube size in the MAFA-ILS method. For cognitive purposes, *SR* values obtained in the MIRLS estimation are presented in the brackets.

Table 1. SRs for the MAFA-ILS method and the MIRLS estimation

PDOP	SR [%]						
	s=10d	s=12d	s=14d	s=16d	s=18d	s=20d	s=22d
2.5	59.25 (64.10)	61.75 (64.70)	62.65 (63.40)	63.95 (64.30)	63.15 (63.40)	63.90 (63.95)	**63.65 (63.65)**
1.7	96.05 (100.00)	98.20 (100.00)	99.30 (100.00)	99.80 (100.00)	99.95 (100.00)	**100.00 (100.00)**	–

The results of above calculations show that, the suggested solution for the minimum size value of appropriate search cube (13) is valid. For both scenarios, the smaller values even proved sufficient. Namely, it is seen that in the MAFA-ILS method, the size of the appropriate search cube were equal here: $s = 1.562$ m ($22d$, $d = 0.071$ m) and $s = 1.140$ m ($20d$, $d = 0.057$ m), for the average and good constellation, respectively. Hence, the suggested formula (13) is a fairly conservative solution.

Conclusions

The MAFA-ILS method is one of the modern method for the precise and real-time GNSS positioning. However, this method still has open non-trivial scientific problems. The determination of the minimum size of appropriate search cube is one of them. This paper is exactly related to this problem.

The paper presents and pre-tests a certain empirical solution. The numerical experiments gave two important conclusions. Firstly, the suggested here formula for the minimum size value of appropriate search cube (13) is valid. Secondly, and more important, there is a relationship between the minimum search cube size and the satellites' constellation geometry. This conclusion is very important for the further research. Because of that, in the future it will be possible to develop a universal analytical formula that will allow to calculate the value of the minimum size of appropriate search cube for any satellites' configuration.

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References

Cai, J.; Grafarend, E.; Hu, C. 2009. The total optimal search criterion in solving the mixed integer linear model with GNSS carrier phase observations, *GPS Solutions* 13: 221–230. https://doi.org/10.1007/s10291-008-0115-y

Cellmer, S. 2011. The real time precise positioning using MAFA method, in *The 8th International Conference Environmental Engineering,* 19–20 May 2011, Vilnius, Lithuania, Vol. III, 1310–1314.

Cellmer, S. 2012. A graphic representation of the necessary condition for the MAFA method, *IEEE Transactions on Geoscience and Remote Sensing* 50(2): 482–488. https://doi.org/10.1109/TGRS.2011.2161321

Cellmer, S. 2013. Search procedure for improving Modified Ambiguity Function Approach, *Survey Review* 45: 380–385. https://doi.org/10.1179/1752270613Y.0000000045

Cellmer, S. 2015. Validation procedure in the Modified Ambiguity Function Approach, *Acta Geodynamica et Geomaterialia* 10: 393–400. https://doi.org/10.13168/AGG.2015.0016

Cellmer, S.; Wielgosz, P.; Rzepecka, Z. 2010. Modified ambiguity function approach for GPS carrier phase positioning, *Journal of Geodesy* 84: 264–275. https://doi.org/10.1007/s00190-009-0364-8

Counselman, C.; Gourevitch, S. 1981. Miniature interferometer terminals for earth surveying: Ambiguity and multipath with global positioning system, *IEEE Transaction on Geoscience and Remote Sensing* 19: 244–252. https://doi.org/10.1109/TGRS.1981.350379

Duchnowski, R.; Wiśniewski, Z. 2014. Estimation of the shift between parameters of functional models of geodetic observations by applying M-split estimation, *Journal of Surveying Engineering* 138: 1–8. https://doi.org/10.1061/(ASCE)SU.1943-5428.0000062

Hofmann-Wallenhof, B.; Lichtenegger, H.; Wasle, E. 2008. *GNSS Global Navigation Satellite System: GPS, GLONASS, Galileo and more*. Wien: Springer.

Kwaśniak, D.; Cellmer, S.; Nowel, K. 2016. Schreiber's differencing scheme applied to carrier phase observations in the MAFA method, in *Proceedings of 2016 Baltic Geodetic Congress (BGC Geomatics)*, 2–4 June 2016, Gdańsk, Poland, 197–204.

Nowel, K. 2015. Robust M-estimation in analysis of control network deformations: classical and new method, *Journal of Surveying Engineering* 141: 1–10. https://doi.org/10.1061/(ASCE)SU.1943-5428.0000144

Nowel, K.; Cellmer, S.; Kwaśniak, D. 2016. A Mixed Integer-Real Least Squares Estimation for precise GNSS positioning using a Modified Ambiguity Function Approach, *GPS Solutions* (under review).

Nowel, K.; Kamiński, W. 2014. Robust estimation of deformation from observation differences for free control networks, *Journal of Geodesy* 88: 749–764. https://doi.org/10.1007/s00190-014-0719-7

Paziewski, J.; Wielgosz, P. 2014. Assessment of GPS+Galileo and multi-frequency Galileo single-epoch precise positioning with network corrections, *GPS Solutions* 18: 571–579. https://doi.org/10.1007/s10291-013-0355-3

Teunissen, P. 1995. The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation, *Journal of Geodesy* 70: 65–82. https://doi.org/10.1007/BF00863419

Teunissen, P. 1999a. The probability distribution of the GPS baseline for a class of integer ambiguity estimators, *Journal of Geodesy* 73: 275–284. https://doi.org/10.1007/s001900050244

Teunissen, P. 1999b. An optimality property of the integer least-squares estimator, *Journal of Geodesy* 73: 587–593. https://doi.org/10.1007/s001900050269

Wiśniewski, Z.; Zienkiewicz, M. 2016. Shift-M-split* estimation in deformation analyses, *Journal of Surveying Engineering* 142: 1–13. https://doi.org/10.1061/(ASCE)SU.1943-5428.0000183