

The Speeds and Accelerations of the Galaxies Movements According to Redshift Measurements

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Abstract. The theoretical presumptions and some experimental calculations to analyse the speeds of the galaxies movements according to redshift measurements applying the Doppler effect are presented. The speed of the galaxy movement is treated as multidimensional continuous value, when values of the speed vector are calculated according to measurements of the redshift parameter z at corresponding moments of the universe lookback time. The most reliable values of the galaxy speeds vector are calculated applying the least square method to the vector of z measurements and including the additional parameters to eliminate the possible systematic errors. The acceleration of the galaxy movement is calculated as a speed fluxion according to period of the adopted redshift signal frequency and as a speed change during the lookback time interval. The expressions of functions of the galaxies speeds and accelerations are received by the polynomial approximation, when values of the polynomial parameters are calculated by the least square method.

Keywords: galaxies: evolution, galaxies: distances and redshifts, galaxies: kinematics and dynamics.

Conference topic: Technologies of geodesy and cadastre.

Introduction

An analyse of the possibilities to estimate the probably existing expansion of the universe according to increasing distances between galaxies is given in this article. Number of authors are analysing theoretical presumptions about the correlation of the Doppler effect and Hubble constant, connecting existing redshift and blueshift of the light spectrum with changes of the distances between galaxies (Jacoby *et al.* 1992; Watkins, Feldman 1995, 2015; Strauss, Willick 1995; Juskiewicz *et al.* 2000; Hudson *et al.* 2004; Bromm, Loeb 2006; Watkins *et al.* 2009; Courtois *et al.* 2012; Davis, Scrimgeour 2014; Whitbourn, Shanks 2014; Springob *et al.* 2014; Hong *et al.* 2014; Zolotov *et al.* 2015; Padmanabhan *et al.* 2015; Moresco 2015; Churazov *et al.* 2015; Agarwal *et al.* 2015; Courteau *et al.* 2015). Also in the literature reader could find wide theoretical speculations about the cosmological origin (genesis) of the redshift radiation, connecting it with photons spreading and with homogeneous and isotropic expansion of universe (Clowes *et al.* 2013; Cassarà *et al.* 2015; Yahya *et al.* 2015; Guglielmo *et al.* 2015, Hagi *et al.* 2015). To explain the redshift phenomena the theory of gravitation and Einstein shift formulae is used:

$$1+z = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}},$$

where z – parameter of the gravitational redshift, G – gravitational constant, M – mass of a body, which creates a gravitational field, r – radial coordinate of the emission source, c – speed of light (Earman, Glymour 1980; Florides 2002).

We are presenting the theoretical skeleton of a model and practical calculations of the changes of the distances between galaxies and their speeds according to changes of the redshift of the light spectrum applying the Doppler effect. One could state, that the cosmic expansion has nothing with Doppler effect. However, the result, received in this research, seems could be interesting to readers.

Theoretical model

To estimate the speeds changes of the galaxies movements we will apply the Doppler effect formulae (Kahmen 1978), when Lorentz factor of relativity is estimated:

$$f_o = f_e \left(1 \pm \frac{c_o}{c_e}\right) \left(1 - \frac{c_o^2}{c_e^2}\right)^{-1/2}, \quad (1)$$

where f_o – frequency of the radiation (light) received (reflected) from the moving body (in our case – galaxy), f_e – frequency of the radiation transmitted from a body, c_o – radial speed or a difference of the transmitter and receiver speeds, c_e – speed of the transmitted radiation in the medium. The sign “+” means, that the distance between transmitter and receiver is decreasing, and “-” – that the distance is increasing.

Let’s write an expression (1) in two forms, when a distance between receiver and transmitter is decreasing and when it is increasing.

In the case, when distance is decreasing we have:

$$\frac{f_o - f_e}{f_e} + 1 = \left(\frac{1 + c_o / c_e}{1 - c_o / c_e} \right)^{1/2} \quad (2)$$

or

$$h^2 = (1 + k) / (1 - k), \quad (3)$$

and

$$k = (h^2 - 1) / (h^2 + 1), \quad (4)$$

In last formulas these notations were used:

$$h = z + 1, \quad (5)$$

$$z = \Delta f_{o,e} / f_e, \quad (6)$$

where

$$\Delta f_{o,e} = f_o - f_e,$$

$$k = c_o / c_e.$$

In the case, when distance between receiver and transmitter is increasing we have:

$$\frac{f_o - f_e}{f_e} + 1 = \left(\frac{1 - c_o / c_e}{1 + c_o / c_e} \right)^{1/2} \quad (7)$$

or

$$h^2 = (1 - k) / (1 + k), \quad (8)$$

and

$$k = (1 - h^2) / (1 + h^2). \quad (9)$$

When a receiver receives the light oscillation from a light source, for example, from the galaxy, when a parameter k shows the speed of the galaxy movement in the units of the light speed in the vacuum, accepting $c_e = 1$.

Modelling results

For modelling we will use the prognostic data of the galaxies redshifts from Pilipenko S. V. paper (Pilipenko 2013). We will reduce the redshift parameter $z' = f_e / f_o - 1$ to expression $z = f_o / f_e - 1$ or $z = (z' + 1)^{-1} - 1$, which are used in formulas (1)–(9).

Now, to relate the galaxies moving speed with the expansion of universe, we will express the redshift parameter z as a function (8) through relative speed parameter k, which shows the relative speed of galaxies against the speed of light c:

$$z = h - 1 = \left(\frac{1 - k}{1 + k} \right)^{1/2} - 1. \quad (10)$$

To process the z data and to eliminate the random errors we will use the least square method. To eliminate the possible systematic errors we will apply the vector δ of the additional parameters.

Using the z data from the different periods of universe existence and by linearising expression (10) we can write the system of equations:

$$\tilde{z}_i = z_i + v_i = -\frac{1}{2} (1 - k_i^2)^{-1/2} \times \left\{ 1 + (1 - k_i)(1 + k_i)^{-1} \tau_j + \delta_i + \bar{h} - 1 \right\}, \quad i = 1 : n, j = 1 : r, \quad (11)$$

where \tilde{z}_i – adjusted values, v_i – corrections of z_i , τ_j – corrections of parameter k_j , \bar{h} – average value of the parameters h_i calculated according to average values k_j , δ_i – systematic errors of z_i . Errors δ_i will be calculated in some equations only.

The adjusted values of the parameter \tilde{k}_j is calculated from the formula

$$\tilde{k}_j = k_j + \tau_j.$$

Using the notation:

$$A_i = -\frac{1}{2} \left(1 - k_i^2\right)^{-1/2} \left\{1 + (1 - k_i)(1 + k_i)^{-1}\right\} \quad (12)$$

and

$$A = (A_1, A_2, \dots, A_n)^T,$$

the system of Eqs (11) could be written in matrixes form

$$V = A\tau + C\delta + L, \quad (13)$$

where V – vector of corrections to z_i ($n \times 1$), A – matrix of coefficients of the corrections ($n \times r$), τ – vector of corrections of the parameters k_i ($r \times 1$), δ – vector of the systematic errors ($s \times 1$), C – matrix of the units in the equations of the systematic errors δ ($n \times s$), $L = (\bar{h} - 1)e - z$ – vector of free parameters ($n \times 1$), s – number of values z_i , for which the systematic errors will be calculated, e – vector of units. Number of values n should be bigger than total number $r + s$ of parameters τ and δ ie $n > r + s$.

In the processing procedures the few modelling variants were used, when the displacement of the main and systematic parameters τ and δ was done by changing their positions in the variants and by using positioning in the different equations of the system (11).

The system of Eqs (12) is solved by least squares method applying condition:

$$\Phi = V^T P V = \min. \quad (14)$$

and solution is

$$\left. \begin{aligned} \frac{\partial \Phi}{\partial \tau} = 2A^T P V = 0 \\ \frac{\partial \Phi}{\partial \delta} = 2C^T P V = 0 \end{aligned} \right\}, \quad (15)$$

where P – matrix ($n \times n$) of weights of the measured values z .

The weights of the measured values z_i are calculated from the formulae:

$$p_{z_i} = \frac{\sigma_0^2}{\sigma_{z_i}^2}, \quad (16)$$

where σ_0 – standard deviation of the free chosen measurements result, weight of which equal to unit ie $p = 1$.

The standard deviations σ_{z_i} could be calculated from formulae:

$$\sigma_{z_i}^2 = \frac{1}{f_{e,i}^2} \sigma_{ob}^2 + \frac{f_{e,i}^2}{f_{e,i}^4} \sigma_e^2 = f_{e,i}^{-2} (1 + h_i^2) \sigma_e^2,$$

where it was accepted $\sigma_{ob} = \sigma_e$; σ_{ob}, σ_e – standard deviations of corresponding frequencies f_o and f_e of received and transmitted oscillations.

Further we have

$$p_{z_i} = \frac{\sigma_0^2 f_{e,i}^2}{(1 + h_i^2) \sigma_e^2} = \frac{1}{1 + h_i^2}, \quad (17)$$

where it was accepted $\sigma_0^2 f_{e,i}^2 = \sigma_e^2$, because value of σ_0 was chosen free, and the frequencies $f_{e,i}$ of the transmitted oscillations are the same.

The system of normal equations looks like:

$$\left. \begin{aligned} N_{11}\tau + N_{12}\delta + \omega_1 = 0 \\ N_{21}\tau + N_{22}\delta + \omega_2 = 0 \end{aligned} \right\}, \quad (18)$$

or

$$N\tau_0 + \omega = 0 \quad (19)$$

where $N_{11} = A^T PA$, $N_{12} = A^T PC$, $N_{21} = C^T PA$, $N_{22} = C^T PC$, $\omega_1 = A^T PL$, $\omega_2 = C^T PL$,
 $N = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix}$, $\tau_0 = \begin{pmatrix} \tau \\ \delta \end{pmatrix}$, $\omega = \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}$.

Solution of the normal equations system (19) is

$$\tau_0 = -N^{-1}\omega. \quad (20)$$

The accuracy of τ_0 is estimated by a covariance matrix K_{τ_0} :

$$K_{\tau_0} = N^{-1}K_{\omega}N^{-1} = \sigma_0^2 N^{-1}. \quad (21)$$

The adjusted values of the vectors \tilde{z} and \tilde{k} could be calculated from formulas

$$\tilde{z} = z + v,$$

and

$$\tilde{k} = k + \tau.$$

An estimation σ'_0 of the standard deviation σ_0 could be calculated from formulae:

$$\sigma'^2_0 = \frac{1}{n-(r+s)} V^T P V. \quad (22)$$

An estimation of the standard deviation was obtained equal to $\sigma'_0 = 0.03z$ unit.

The initial z_i and adjusted \tilde{z}_i values are given in Table 1.

Table 1. The measured and adjusted values of the parameter z

Number of z value	Initial data of the parameter z'	Lookback time of galaxies, billion (10^9) years	Initial data of the parameter z	Data of adjusted values \tilde{z}
	0.050	0.600	-0.048	-0.048
	0.100	1.300	-0.091	-0.091
	0.300	3.600	-0.231	-0.231
	0.500	5.200	-0.333	-0.333
	0.800	7.100	-0.444	-0.444
	1.000	8.100	-0.500	-0.500
	1.500	9.600	-0.600	-0.670
	2.000	10.600	-0.667	-0.695
	2.500	11.100	-0.714	-0.717
	3.000	11.700	-0.750	-0.783
	3.500	12.050	-0.778	-0.790
	4.000	12.300	-0.800	-0.796
	4.500	12.500	-0.818	-0.803
	5.000	12.650	-0.833	-0.809
	5.500	12.800	-0.846	-0.816
	6.000	12.900	-0.857	-0.823
	7.000	13.050	-0.875	-0.836
	8.000	13.200	-0.889	-0.850
	9.000	13.300	-0.900	-0.864
	10.000	13.350	-0.909	-0.878
	11.000	13.400	-0.917	-0.892
	12.000	13.460	-0.923	-0.906
	13.000	13.500	-0.929	-0.920
	14.000	13.520	-0.933	-0.934
	16.000	13.580	-0.941	-0.962
	18.000	13.610	-0.947	-0.990
	20.000	13.640	-0.952	-1.018

The speeds of galaxies k_i , calculated according to z_i , and speeds \tilde{k}_i , calculated according to \tilde{z}_i are shown in Table 2.

Table 2. Data of the galaxies speeds

Number of z value	Speeds according to initial z data	Speeds according to adjusted \tilde{z} data
	0.0488	0.0488
	0.0950	0.0950
	0.2565	0.2565
	0.3846	0.3846
	0.5283	0.5283
	0.6000	0.6000
	0.7241	0.8036
	0.8000	0.8299
	0.8491	0.8515
	0.8824	0.9104
	0.9059	0.9152
	0.9231	0.9201
	0.9360	0.9250
	0.9459	0.9297
	0.9538	0.9344
	0.9600	0.9390
	0.9692	0.9478
	0.9756	0.9560
	0.9802	0.9636
	0.9836	0.9705
	0.9862	0.9768
	0.9882	0.9823
	0.9898	0.9872
	0.9912	0.9912
	0.9931	0.9971
	0.9945	0.9998
	0.9955	0.9994

The curve of the galaxies speeds according to z_i and its approximation by the least squares method is presented in Figure 1, and the curve of the galaxies speeds according to \tilde{z}_i and its approximation by the least squares method is presented in Figure 2.

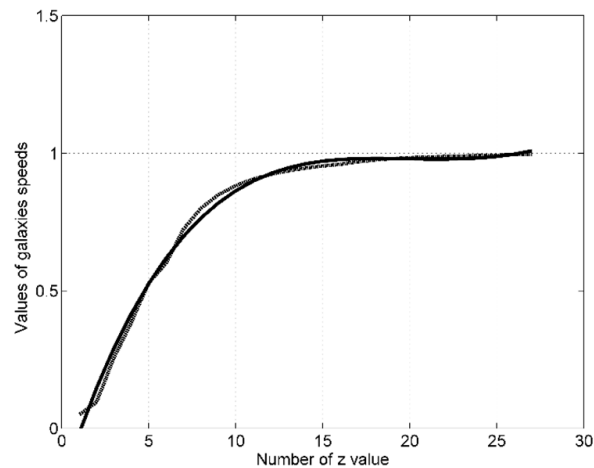


Fig. 1. The approximation function of the galaxies speeds according to initial z data

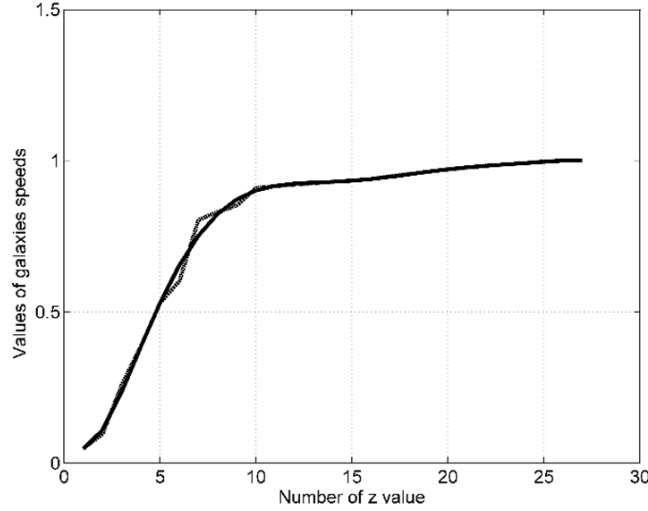


Fig. 2. The approximation function of the adjusted galaxies speeds

To understand the changes of the galaxies movements related to universe expansion it is possible by analysing the accelerations of the galaxies movements. In the further calculations we will analyse the accelerations of the galaxies in two variants: by detecting the acceleration from the theoretical formulae as the partial fluxion according to accepted light oscillations period and by detecting the acceleration as the change of the galaxies speed for the unit of the lookback time interval of the universe.

Using formulae (9):

$$k = (1 - h^2)(1 + h^2)^{-1}$$

we have expression for the acceleration a_i

$$\begin{aligned} a_i &= \frac{\partial k_i}{\partial T_{o,i}} = \frac{\partial k_i}{\partial h_i} \frac{\partial h_i}{\partial T_{o,i}} = -2h_i (1 + h_i^2)^{-1} \times \left\{ 1 + (1 - h_i^2)(1 + h_i^2)^{-1} \right\} \frac{\partial h_i}{\partial T_{o,i}} = \\ &= -4h_i (1 + h_i^2)^{-2} \frac{\partial h_i}{\partial T_{o,i}} = 4h_i^2 (1 + h_i^2)^{-2} f_{o,i} = 4h_i^3 (1 + h_i^2)^{-2} f_{e,i}, \end{aligned} \quad (23)$$

where a_i – accelerations, $h_i = z_i + 1 = f_{0,i} / f_{e,i} = T_{e,i} / T_{0,i}$; $T_{e,i}, T_{0,i}$ – periods of the corresponding light oscillations, $i = 1 : n$ (n – number of used lookback time intervals).

The formulae to calculate the accelerations using the period T_e of the transmitted light oscillations looks like

$$a_i = 4h_i^3 (1 + h_i^2)^{-2} f_{e,i}. \quad (24)$$

The formulae to calculate the accelerations using the period T_0 of the received light oscillations looks like

$$a_i = 4h_i^2 (1 + h_i^2)^{-2} f_{o,i}. \quad (25)$$

The acceleration, applying the change of the galaxies speed for the unit of the lookback time interval of the universe, could be calculated from formulae:

$$\bar{a}_i = \frac{\delta k_i}{\delta t_i}, \quad (26)$$

where $\delta k_i = k_i - k_1$, $\delta t_i = t_i - t_1$ – lookback time interval.

The accelerations of galaxies, calculated according to z_i , and accelerations, calculated according to \bar{z}_i are shown in Table 3. Also accelerations, calculated from theoretical formula using accepted light oscillations period T_0 , applying two different parameters – the oscillations periods T_0 and T_e of the received and transmitted light.

Table 3. Data of the galaxies accelerations

Number of z value	Accelerations calculated according to initial z data	Accelerations calculated according to initial \bar{z} data	Accelerations calculated by theoretical formulae using the period T_e of the transmitted light oscillations	Accelerations calculated by theoretical formulae using the period T_0 of the received light oscillations
	0.0661	0.0661	0.9501	0.9976
	0.0693	0.0693	0.9009	0.9910
	0.0730	0.0730	0.7186	0.9342
	0.0738	0.0738	0.5680	0.8521
	0.0735	0.0735	0.4005	0.7209
	0.0750	0.0839	0.3200	0.6400
	0.0751	0.0781	0.1902	0.4756
	0.0762	0.0765	0.1200	0.3600
	0.0751	0.0776	0.0797	0.2791
	0.0749	0.0757	0.0554	0.2215
	0.0747	0.0745	0.0399	0.1794
	0.0746	0.0736	0.0296	0.1479
	0.0745	0.0731	0.0225	0.1239
	0.0742	0.0726	0.0175	0.1052
	0.0741	0.0724	0.0139	0.0903
	0.0739	0.0722	0.0112	0.0784
	0.0736	0.0720	0.0076	0.0606
	0.0733	0.0720	0.0054	0.0482
	0.0733	0.0723	0.0039	0.0392
	0.0732	0.0725	0.0030	0.0325
	0.0731	0.0726	0.0023	0.0274
	0.0730	0.0727	0.0018	0.0234
	0.0729	0.0729	0.0014	0.0202
	0.0728	0.0731	0.0012	0.0176
	0.0727	0.0731	0.0008	0.0137
	0.0726	0.0729	0.0006	0.0110
	0.0726	0.0729	0.0004	0.0090

The curve of the galaxies accelerations according to z_i and its approximation by the least squares method is presented in Figures 3 and 4.

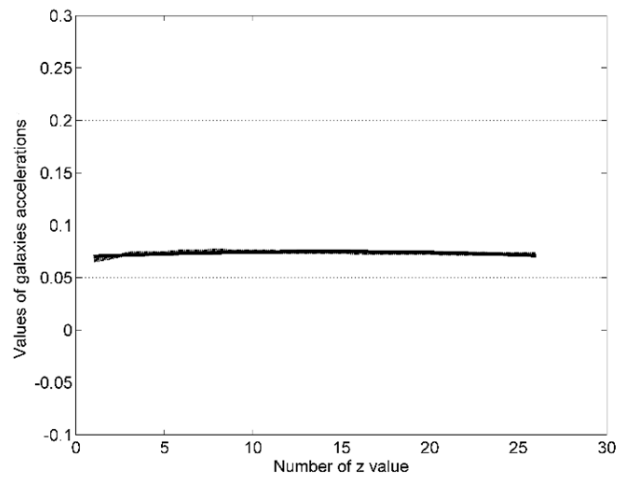


Fig. 3. The approximation function of the galaxies accelerations according to initial z data

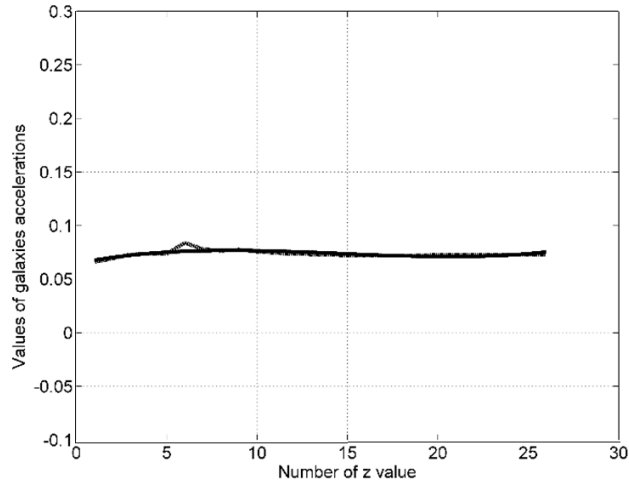


Fig. 4. The approximation function of the galaxies adjusted accelerations

The accelerations from the theoretical formulae as the partial fluxion according to accepted light oscillations period T_0 calculated in two variants, when the parameters of the received and transmitted light oscillations were used, are shown in Figures 5 and 6.

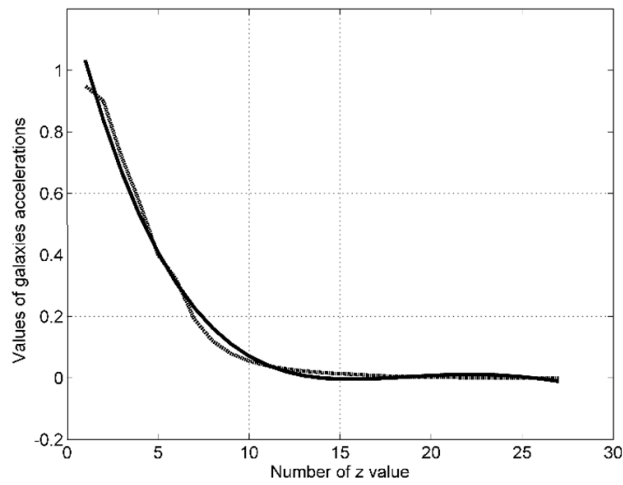


Fig. 5. The approximation function of the galaxies accelerations according to the period T_e of the transmitted light oscillations

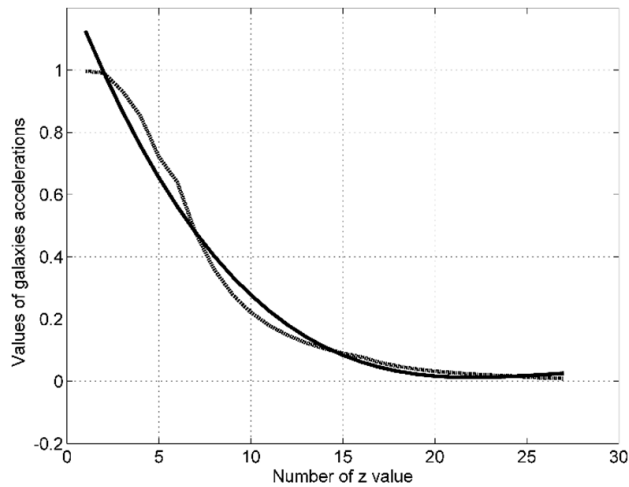


Fig. 6. The approximation function of the galaxies accelerations according to the period T_0 of the received light oscillations

Conclusions

1. The skeleton of a model for the detection of the universe expansion using Doppler effect with redshift parameter z , when all data of the universe life period is processed by the least squares method and the adjusted values \tilde{z} and the parameter values \tilde{k} of the universe expansion speed are calculated, is suggested. That ensures the more reliable values of estimated parameters in the probability sense. The detected value of the universe expansion speed parameter k is approaching to unit, when $|z| > 0.9231$. That corresponds to the period of the universe expansion $t \rightarrow 13.5$ mlrd. years. As a unit of weight of the parameter k the speed of light in vacuum was used, when $c = 1$.

2. The skeleton of a model of the universe expansion acceleration, when acceleration is calculated as the partial fluxion according to accepted light oscillations period T_0 calculated in two variants, applying two different parameters – the oscillations periods T_0 and T_e of the received and transmitted light, is presented. In both cases the results are very close, and the value of the acceleration a is approaching to zero, when $|z| > 0.9412$.

3. The acceleration a of the universe expansion calculated according the formulas (24) and (25), when the initial z and adjusted \tilde{z} values were used, is stable (in the limits of model accuracy) and is equal to $a \rightarrow 0.073c / s^2$.

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