

The Impact of Load Bearing Capacity of Airfield Pavement Structures on the Air Traffic Safety

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Abstract. Airfield pavement is a marked and appropriately prepared surface of an airfield functional element that performs a definite function in aircraft operations. The structure of airfield pavement is most often composed of a set of layers whose task is to absorb and transfer loads coming from moving aircraft onto the ground in a way that ensures its definite durability. Structures of airfield pavements are designed for a definite exploitation period on the assumption of predicted volume and structure of the air traffic. Safety of air operations conducted by aircrafts on airfield pavements depends mainly on the state of bearing capacity of their construction. Due to the above, control tests of bearing capacity shall be periodically conducted, since information regarding the current state of an airfield pavement constitutes the basis for decisions concerning the types of aircrafts permitted to land and take off, traffic volume and dates of starting renovation or modernization works. In addition to loads generated by aircraft, on the condition of airfield pavement load bearing capacity is influenced by many external factors, including weather conditions.

The ACN-PCN non-destructive method is currently used in the assessment of airfield bearing capacity, which has been introduced by ICAO (ICAO 2013). According to its assumptions, the airfield construction bearing capacity may be expressed in PCN or permissible number of air operations. The fundamental problem by measuring airfield pavements is to assume the correct computational model of a structure, which describes the way of cooperation and mechanical properties of individual layers.

This paper contains the way of assessing and description of PCN as well as presentation of the possibility of expressing bearing capacity results by determination of permissible number of aircraft operations. There is also interrelation between PCN and the permissible number of aircraft operations presented in a graphic way.

Keywords: load bearing capacity assessment, concrete airfield pavements, ACN-PCN method.

Conference topic: sustainable urban development.

Introduction

The problem of safety in the air transport will be topical as long as aviation exists. Ensuring safety was, has been, and will be one of the most important problems in functioning and development of aviation.

Safety during air operations is affected by all the elements involved in this process, which can be included in three groups, namely; human, aircraft, and environment (including airport pavements). The airport pavements, in terms of safety, are characterised by identifying their technical condition.

The first and most important operation parameter of the airport pavements is the load capacity, which clearly determines the ability of a considered structure to transfer loads safely from aircraft at a certain time.

Under Polish weather conditions, the most often and used design solution is concrete airport pavements.

The state of load capacity of the airport pavements depends not only on the loads generated by the aeroplanes, but also on many external factors, including weather conditions. The mechanical properties of the concrete, of which the airport pavement was made, and the type of ground subsoil directly below the assessed pavement structure have a very significant impact on the final load capacity result. Therefore, the most important factors affecting the concrete airport pavements load capacity can include (Blacha, Wesołowski 2016):

- number of air operations (loads of the aircraft) taking place or planned on the said pavement;
- cross-section of pavement structure – thickness of particular structural layers;
- concrete bending strength;
- concrete modulus of elasticity;
- Poisson’s coefficient of the pavement and subsoil’s structural layers;
- type, density, and moisture of the ground subsoil;
- ground subsoil load capacity;
- temperature during executed field tests.

While the first three parameters can be determined as a fixed or stable for a short period of time, the parameters of the ground subsoil may, however, vary depending on the existing weather conditions. In the case of the airport pavements made of cement concrete, the impact of the phenomenon of concrete slabs’ deformation under the influence

of temperature must also be taken into account. The impact of the ground subsoil on the load capacity of the pavement is caused by changes in geotechnical parameters of soil depending on its moisture.

Due to safety of the pavement usage, it is normally assumed that load capacity tests should be executed in spring or late autumn, and should not be executed in winter. The full analysis of the airport requires identification of physico-chemical parameters of the material of its particular structural layers and ground subsoil. The identification should be executed in a way that takes into account the actual conditions of the pavement's operation.

In the literature and in the documents of aviation (FAA 1997, 2004; ICAO 2013) are presents the basic assumptions of the ACN-PCN method to assessment of capacity of airfield pavements structures. Unfortunately, they lack a detailed methodology for conducting field research on the measurement of deflection and download *in situ* samples of the materials from which made the construction assessed for comprehensive assessment the real state of pavement load capacity of airfield pavements expressed in terms of indicator of PCN. Therefore, Airfield Pavement Division of Air Force Institute of Technology (AFIT) developed and implemented for use in the Polish Armed Forces standard defensive NO-17-A500: 2016 *Airfield and road pavements – Load capacity testing*, which specifies the requirements for the testing load capacity of airfield pavements.

Airport pavement structures

Many years of experience of the Airport Department of the Air Force Institute of Technology (AFIT) in terms of conducted tests of airport pavements load capacity on military facilities and Airports confirms that there are three basic types of pavement structures in Poland:

- rigid (elastic) pavements made of cement concrete;
- flexible pavements made of asphalt concrete;
- complex (elastic-flexible) pavements, in which the rigid structure is reinforced with a layer of asphalt concrete.

The proper identification of the structure has a significant impact on determining the airport pavements load capacity because the manner of transferring the load by the aircraft to the ground subsoil is dependent on the type of airport pavement structure. Depending on the type and the manner of operation, to express the nature of the aircraft's impact on the pavement, appropriate mathematical models, which are presented in the chapter 3, are applied.

ACN-PCN method of assessment of airport pavements load capacity

In this method, the Aircraft Classification Number (ACN) expresses the relative impact of the aeroplane on the airport pavement when the standard load capacity of the ground subsoil is determined. ICAO defined the procedure for determining the ACN, which assumes that the standard volumes in the process of its calculation are:

- the pressure in the tyre of a single main leg wheel equal to 1.25 MPa;
- allowable bending stresses in a concrete plate (for rigid pavements) equal to 2.75 MPa; allowable number of loads in the case of flexible pavements;
- load capacity of the ground subsoil, described with the subsoil reaction coefficient k for the rigid pavements and with the CBR index for flexible pavements.

The ACN is specified with the formula:

$$ACN = 2 \cdot P_r, \quad (1)$$

where: P_r – equivalent load in thousands of kilograms of such a value that the pavement's thickness necessary for its transfer is equal to the thickness defined as for the actual load: $P_r = \pi \cdot q \cdot a^2$; q – uniform load with an intensity of 1.25 MPa, distributed on the circular area of radius a .

The load capacity of the rigid pavement is influenced by: shape and dimensions of concrete plates, the manner of their adhesion to the ground subsoil and the concrete's strength parameters. The load capacity of the flexible pavement structure depends on the number and thickness of layers, as well as the physico-mechanical properties of these layers' materials. Rigidity moduli of particular structural layers, the corresponding Poisson's coefficient values and inter-layer connections' condition, cracks in the pavement layers, water penetrating the structure and the ground subsoil, as well as the temperature of the asphalt layers are particularly significant. The load capacity of the ground subsoil is an important factor in determining the ability of the pavement to take the loads. It is known that the load distributed throughout the pavement structure influence a narrower area in the case of high load capacity of the subsoil than in the case, when the same structure is supported on a subsoil with a low load capacity. This means a significant reduction (in the first of the mentioned cases) in complex influence of adjacent wheels of the aeroplane legs. This is also why, in order to determine the impact of the aeroplane on the pavement with the ACN, the distribution of the subsoil load capacity into four categories: high, medium, low, and very low was adopted in the discussed method. The aeroplane ACNs are determined taking into account the standard values of the subsoil's load capacity.

For each aircraft, the ACN is a set of numbers depending on the type of airport pavement's structure (rigid and flexible) and the load capacity of the ground subsoil. Tables containing the ACNs' values of the most commonly used types of aircraft are given in (NO-17-A500:2016).

The Pavement Classification Number (PCN) expresses the load capacity of the airport pavement for a limited number of aircraft's travels with the CAN = PCN. It is equivalent to 1/500 of the allowable load (expressed in kilograms of weight) applied to the pavement via a single wheel with a standard pressure equal to 1.25 MPa.

In assessing the load capacity of the airport pavements, in many countries, the back calculation procedures are applied (FAA 1997). In Poland, the methods adopted from abroad are applied in testing the load capacity of the airport pavements. For flexible pavements, the calculation model is the equivalent elastic system of the double-layer half-space. It is recommended to determine modules for the subsoil by tests using measuring plates directly on the ground or in a laboratory, and the layers' modules are determined in laboratory tests. For the rigid pavements, the model is a plate on the Winkler subsoil. It is recommended to determine the coefficient of ground reaction during direct tests on the ground. Currently, in order to assess the carrying capacity of the airport pavements a Heavy Weight Deflectometer (HWD) is applied. In order to assess the load capacity of the airport pavement using the HWD, a measured deflection bowl of the tested structure is used. On the basis of the deflection bowl and knowledge of the thickness of the structural layers as well as characteristics of the materials of which they are made, moduli of elasticity of the particular layers are determined (Ullidtz 1987). The result of measuring the pavement by means of the HWD are the envelopes of elastic deflections' maximum values measured by all the geophones. This set of values is defined as the deflection bowl and is presented in Figure 1.

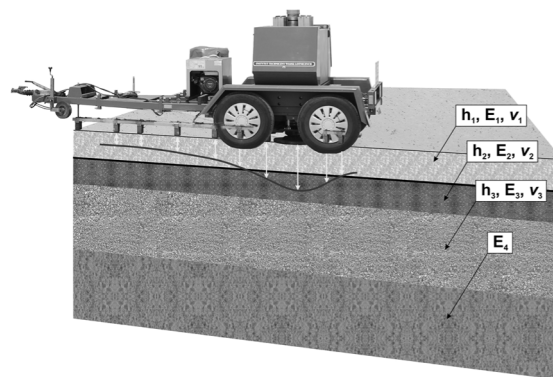


Fig. 1. View of a deflection bowl of measurement with the HWD

The size of deflections in the entire bowl is a relationship, which is described by the following formula:

$$U_i = f(h, E, \nu), \quad (2)$$

where: U_i – deflection value of the test surface in the point i ; f – function relation of the components; h – thickness of the particular pavement structural layers; E – modulus of elasticity of the particular structural layers of the pavement and subsoil; ν – Poisson's coefficient of the pavement and subsoil's structural layers.

Distribution of deflections of the tested pavement changes with the change in the thickness of the particular structural layers, their stiffness, and the Poisson's coefficient. The pavement's subsoil rigidity has the biggest influence on the shape of the entire deflection bowl. The changes in the ground subsoil's rigidity cause shifting the entire deflection bowl upwards (higher subsoil's rigidity) or downwards (lower subsoil's rigidity). Therefore, they have a very significant impact on the deflection values at all measurement points. However, the changes in the rigidity of the upper structural layers of the pavement and substructure change the bowl's shape only at a certain distance from the centre of the applied load. The changes in the upper pavement layers' thickness show a trend similar to the change in this layer's rigidity. The effect of Poisson's coefficient on the size and shape of the deflection bowl was the subject of many analyses. The coefficient's value, depending on the material type in the layer, may be in the range from 0.2 to 0.5. The differentiation of this parameter in the pavement structure leads to similar effects, which have been identified at changing the stiffness and thickness of the layers. In addition, conclusions regarding the effect of the Poisson's coefficient on the tested pavement's deflection result directly from the solution of the problem formulated for the half-space. In the half-space, the deflections are proportional to a known formula in the form (the formula below), which shows that the Poisson's coefficient has little effect on the deflection values (Lybas, Ruth 1982).

$$u = \frac{2 \cdot (1 - \nu)^2 \cdot p \cdot r_0}{E} \cdot u_z(r, z), \quad (3)$$

where: u – deflection on the pavement; ν – Poisson’s coefficient; p – contact pressure; r_0 – radius of the contact area; E – modulus of elasticity; $u_z(r, z)$ – function depending on the spatial coordinates only.

On the basis of the recorded values of the airport pavement’s deflection, the materials’ moduli of elasticity of the particular layers using an iterative comparison of the measured deflections and the theoretical deflections so that the function F has a minimum value. For this purpose, the following relationship is used:

$$F = \sum_{j=1}^k (w_j - u_j)^2, \quad (4)$$

where: F – approximation function of actual and theoretical values; w_j – calculated pavement deflections at a distance r from the centre of a loading plate; u_j – measured pavement deflections at a distance r from the centre of a loading plate; k – number of geophones (measuring sensors describing the deflection bowl) usually 9.

The results can be presented in the form of deflections, deformation moduli, substitute moduli or as the pavement load capacity in line with the ACN-PCN method’s assumptions. The surface moduli, depending on the geophones’ distance from the centre of the loading plate, are determined using the following formulas:

$$E_0(0) = \frac{2 \cdot (1 - \nu^2) \cdot q \cdot a}{u(0)}; \quad (5)$$

$$E_0(r) = \frac{(1 - \nu^2) \cdot q \cdot a^2}{r \cdot u(r)}, \quad (6)$$

where: $E_0(0)$ – surface modulus under the loading plate; $E_0(r)$ – surface modulus at the distance r from the centre of the loading plate; E_z – substitute modulus of the tested pavement; a – plate radius; ν – Poisson’s coefficient; u – deflection in the tested point (0 – under the loading plate); q – stress under the loading plate.

In order to determine the estimated substitute modulus of the tested airport pavement’s structure, the shortened version of the above formulas is used:

$$E_z = \frac{2 \cdot q \cdot a}{u(0)}. \quad (7)$$

Figure 2 shows the HWD used by AFIT for measuring the elastic deflections of the airport pavements.



Fig. 2. HWD when measuring the airport pavement deflections

In the ACN-PCN method, the full information about the airport pavement load capacity should contain the following data: the PCN, pavement structure type, category of subsoil load capacity, category of tyre pressure, used assessment method (ICAO 2013). This information is presented with a group of symbols, as discussed in subsection 4.1, Table 1.

Calculation models of airport pavements structure

The main problem when dimensioning the airport pavements is the adoption of a structure’s calculation model describing the mechanical properties of the particular layers. It is important that the used model, characterised by the given parameters, behaves in a manner as consistent with behaviour of the actual layers, for the description of which it was applied, as possible under operating pressure (Odemark 1949). Together with the development of new technologies and computer techniques over the past several dozen years in the methods of pavement dimensioning, a constant evolution of the models of airport pavement structures’ is observed. Models more complex in terms of mathematical notation are more and more often applied but, at the same time, are closer and closer to the actual behaviour of the structure.

Depending on the type and the manner of the airport pavement and the manner of its operation, to express the nature of the aircraft’s impact on the pavement, the following mathematical models are applied:

- the model of a plate with finite dimensions in a plan, located on the Winkler substrate – for a rigid pavement;
- the model of elastic layered half-space – for a flexible pavement.

The solution of the plate with finite dimensions in a plan's model was developed by Westergaard (Westergaard 1943). The plate, in this model, is described with the Young's modulus E , the Poisson's coefficient ν , and thickness h . The subsoil was, in turn, described with the subsoil reaction coefficient k . The solution of the model of the layered half-space has been developed by, among others, Burmister, Kogan, Nowotny, and Hanuška. In this model, the layers and the subsoil are described with moduli of elasticity E_i , Poisson's coefficients ν_i , and thicknesses h_i . For an elastic half-space loaded on the surface (a flexible pavement), the Boussinesque's solution is known. For the pavement with a complex structure, the analysis must be conducted and the dominating element must be determined. The complex pavements must be qualified to either a rigid or flexible pavements group. In doubtful cases, they are classified as flexible ones.

Model of airport pavement on the Winkler's substrate

The Westergaard's model, which is the most commonly used in the world airport technique, describes the airport pavement of the rigid structure that is in the form of plates with finite dimensions in a plan lying on the inertialess Winkler's substrate. Westergaard, who published his theory of cement concrete pavements' design for the first time in 1927, considered "quarter-infinite" plates, taking into account the three most characteristic locations of the load modelling the wheels' pressure, namely at the corner, in the middle, and at the edge of the plate. The above-mentioned plate loads cases are shown in Figure 3.

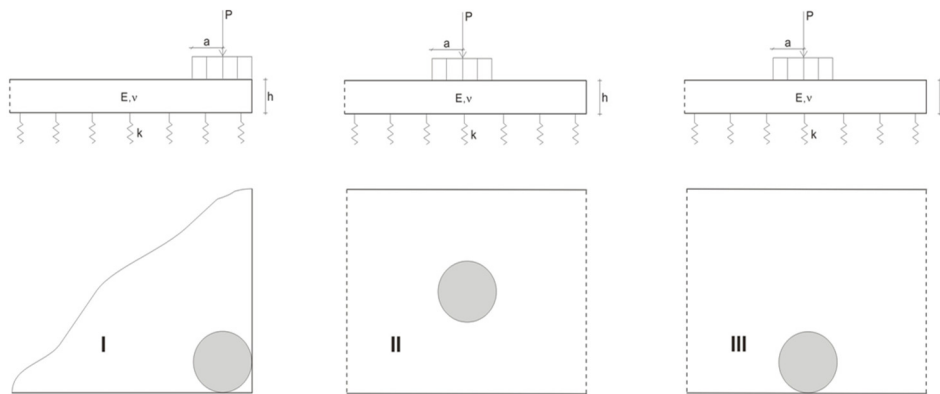


Fig. 3. Calculation scheme according to the Westergaard's model

The formulas deduced by him, which describe the state of maximum bending stresses in the plate for the enlisted load cases, are in the following form:

- for the location I:

$$\sigma_r = \frac{3P}{h^2} \left[1 - \left(\frac{a\sqrt{2}}{l} \right)^{0.6} \right]; \quad (8)$$

- for the location II:

$$\sigma_r = 0.275 \frac{P}{h^2} (1 + \nu) \left[4 \log \left(\frac{l}{b} \right) + 1.069 \right]; \quad (9)$$

- for the location III:

$$\sigma_r = 0.529 (1 + 0.540 \nu) \frac{P}{h^2} \left[4 \log \left(\frac{l}{b} \right) + 0.359 \right], \quad (10)$$

where: P – plate load [kN]; h – plate thickness [m]; ν – Poisson's coefficient; l – radius of the plate relative rigidity; E – modulus of plate elasticity [MPa]; k – substrate reaction coefficient [MPa/m]; a – radius of tyre contact with pavement [m]; b – equivalent radius taking into consideration the load distribution in the plate lower part [m];

$$l = \sqrt[4]{\frac{E \cdot h^3}{12 \cdot (1 - \nu^2) \cdot k}}; \quad (11)$$

$$b = \sqrt{(1.6a^2 + h^2)} - 0.675h, \quad (12)$$

when $a < 1.724h$, $b = a$, when $a > 1.724h$.

Airport pavement model on a layered elastic half-space

In the model of airport pavement on a layered elastic half-space, each layer has unlimited dimension in a horizontal plane, and is characterised by the thickness h_i , modulus of elasticity E_i , and the Poisson's coefficient ν_i . These layers are placed on an elastic half-space (subsoil), which is described with modulus of elasticity E_1 and the Poisson's coefficient ν_1 and has an unlimited dimension in the horizontal and vertical plane, that is $h_1 = \infty$. This model shown in detail in Figure 4.

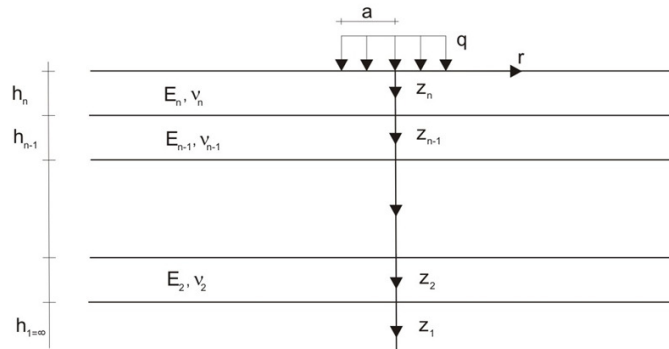


Fig. 4. Model of elastic layered half-space

The model in the form of a layered elastic half-space is the one of the pavement models that are more adequate to reality and is used in the airport technique more and more often. It is used in, among others, methods for reinforcements dimensioning and design: the FAA, the Shell, and the Czechoslovak method. This model also used by Szydło, Pilujski, and Pownug in the issue of identification of airport pavements with the static method.

The described models serve for description of static phenomena. Emergence of new methods in the problem of identifying the pavement load capacity as well as impact and harmonic tests inspired creation of dynamic models. In the previous practice, the impact of wave phenomena on the durability of the pavement structure has been usually omitted. This was due to both the underestimation of this impact and the difficulties of calculation nature. In the actual layered systems, there are complex dynamic phenomena. Among them, the formation of various types of waves can be extracted: transverse waves, longitudinal waves, Rayleigh surface waves, and Love waves. These waves, after reaching the boundary of layers of different physical properties, are refracted and reflected, resulting in the emergence of new disturbances. Therefore, this situation makes it necessary to choose the intermediate path, which involves finding a relationship between the dynamic tests results and the static tests results, on the basis of which the system parameters can already be identified using one of the described inertialess models.

Presentation of the load capacity results

Airport pavements' load capacity expressed with the PCN index

According to the adopted findings, in the ACN-PCN method, the airport pavement load capacity is described by a group of symbols representing various structure parameters and informing about the method of determining the PCN, e.g. PCN 48/R/B/X/T. The exemplary notation indicates the rigid pavement (R) on the ground subsoil with an average load capacity (B) with the surface layer able to take the pressure of up to 1.5 MPa (X). The PCN index was determined with the technical method (T) using the impact deflectometer. Therefore, the aircraft can use such an airport pavement without restriction, whose ACN is lower than an example PCN equal to 48. For example, the ACN for the aeroplane of Airbus A320-200 class is 46, so it can safely perform take-offs and landings on the airport pavement shown in the example. The detailed manner of interpretation of the above-mentioned notation is shown in Table 1.

Verifying whether a given aeroplane can operate safely at a given airport involves, therefore, a comparison of the pavement PCN for various functional elements of the airport and the aeroplane ACN. It should be emphasised that the ICAO has introduced in Annex 14, appendix A, section 19 limitations in traffic of the aircraft exceeding the load capacity (causing the overload) of a given pavement when $ACN > PCN$.

Table 1. PCN index interpretation manner

1	Dimensionless PCN				
2	Pavement type	R	Rigid		
		F	Flexible		
3	Soil category (for rigid pavements – k , for flexible pavements – CBR)	A	high load capacity	$k > 120 \text{ MN/m}^3$	$CBR > 13$
		B	medium load capacity	$60 - 120 \text{ MN/m}^3$	8–13
		C	low load capacity	$25 - 60 \text{ MN/m}^3$	4–8
		D	very low load capacity	$k < 25 \text{ MN/m}^3$	$CBR < 4$
4	Allowable pressure in aeroplane tyres	W	no limitations		
		X	medium up to 1.5 MPa		
		Y	low up to 1.0 MPa		
		Z	very low up to 0.5 MPa		
5	Assessment method	T	technical method		
		U	experimental method		

Load capacity of airport pavement expressed with allowable air operations number

The analysis of load capacity of airport pavements can also be conducted for the predetermined PCN index. In this case, as a load capacity result, the number of allowable air operations, which is determined for a specified load repetitions number N , is given. The number of allowable repetitions is calculated according to the adopted calculation model of the assessed airport pavement structure. For the rigid surfaces, made of cement concrete, the following formula resulting from the allowable bending stresses criterion is applied:

$$N = \left[\frac{R_{zg}}{\sigma} \times \left(\frac{E}{30\,000} \right)^{1.3} \right]^{(1/0.233)} \times 10^4, \tag{13}$$

where: R_{zg} – concrete bending tensile strength [MPa]; σ – bending tensile stresses in the lower concrete plate part [MPa]; E – modulus of concrete elasticity [MPa].

Whereas for the flexible pavements made of asphalt concrete, the formula given below resulting from the allowable deformations criterion must be applied:

$$N = \left[\frac{R_{zg}}{\sigma} \times \left(\frac{E}{160} \right)^{1.23} \right]^{(1/0.173)} \times 10^4, \tag{14}$$

where: R_{zg} – concrete tensile strength when bending the subsoil [MPa]; σ – bending tensile stresses determined for the subsoil [MPa]; E – modulus of concrete rigidity [MPa].

The load repetitions number N specified in this way for the assessed airport pavement is used to determine the allowable air operations number. The values of the applied conversion coefficients, depending on the aircraft’s main landing gear and the pressure in its tyres are given in (Directorate of Civil Engineering ... 1989). On this basis, the relationship of the PCN index and the allowable air operations number can be specified, which was shown graphically in Figure 5 (Blacha, Wesołowski 2014).

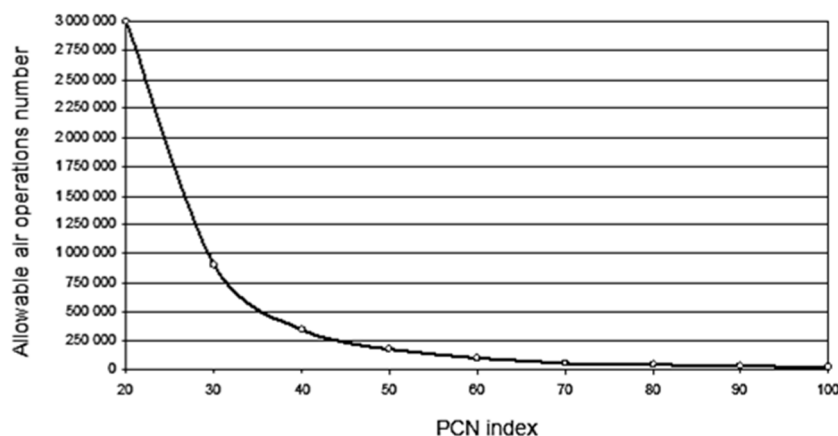


Fig. 5. Relationship of the PCN index and the allowable air operations number

Conclusions

Load capacity is the one of the most important parameters affecting the safety of performing air operations. The method of assessing the airport pavements load capacity (ACN-PCN) is a non-destructive, dynamic, and very effective method. It also enables to classify the pavements load capacity on the basis on the rheological tests of layers' material and, at the same time, precise predicting the pavements operating time with known prediction of the land traffic of the aircraft. Load capacity of the airport pavements can be expressed with the PCN index or the allowable air operations number. In both cases, the allowable load repetitions number N, which impacts directly the load capacity value, is important. Attention must be paid to the fact that the allowable load repetitions number N is a limited number. Therefore, it is inappropriate to declare, when determining the PCN index for the airport pavements that the load repetitions number is unlimited or the allowable air operations number is unlimited. In Poland, it is adopted that the airport pavements with flexible structure are designed for 20 years of operation, while the rigid pavements – for 30 years. The methods of designing the airport pavements applied worldwide taking into account the operation periods given above assume three categories of air traffic intensity, for which the nominal air operations numbers were specified (Directorate of Civil Engineering ... 1989): low – 10 000 air operations; medium – 100 000 air operations and high – 250 000 air operations.

Therefore, it must be stated that there is a close relationship of the load capacity PCN index and the allowable air operations number when determining the airport pavements load capacity with the ACN-PCN method.

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